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OPTIMAL DISTRIBUTION OF INCOME FOUND BY EVOLUTIONARY COMPUTATION

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Abstract. *In this research we tried to answer the question: How to optimize the total production of economy. To find an answer we used as ingredients two concepts: 1) The capital, in control of the decisions in production, will want to maximize total production since the profit is empirically always 5% of production. 2) A worker will be incentivated to work if he sees it pays off. When his neighbor below him earns less and above him earns more he will work harder and produce more. The productivity of the worker is proportional to this 'derivative' in the income curve. (Note: a worker's salary is not assumed necessarily proportional to his productivity)*

We used these two ingredients in evolutionary computation. Starting with a distribution, we make random small changes to it and if the total production increases, the capital will decide to implement these changes. This procedure is repeated until the distribution is stable.

The results of our computations are:
- *There are a few people that work a little, and get no income*
- *There are many who do nothing and get a meager salary*
- *There are few who work tremendously and get the lion's share of income*

1 INTRODUCTION

One of the prime goals of society is to organize economy in such a way that it optimizes the production of goods, assuming that wealth is equal to the consumption of goods and consumption makes people happy. In fact, it is the prime ingredient of the philosophical school of Utilitarianism of Jeremy Bentham (1748-1832) and John Stuart Mill (1806-1873): "The greatest good for the greatest number"[1].

Slightly changing this goal, we tried to answer the question: How to optimize the *total* production. To find an answer we used as ingredients two concepts. Either of the two can be questioned, but seem quite reasonable:

1. The capital, in control of the decisions in production, will want to maximize total production since the profit is empirically always 5% of production[2]; increasing production will increase profit.

2. A worker will be incentivated to work if he sees it pays off. When his neighbor below him earns less and above him earns more he will work harder and produce more. The productivity of the worker is proportional to this 'derivative' in the income curve.

(Note: a worker's salary is not assumed necessarily proportional to his productivity)

2 RESULTS AND DISCUSSION

We used these two ingredients in an evolutionary computation, or 'molecular dynamics', that works in the following way. Starting with a distribution, we make random small changes to it and if the total production increases (if the 'energy' is lower), the capital will decide to implement these changes; in the molecular dynamics simulation we keep them. This procedure is repeated until the distribution is stable. We call this the final state.

The distribution of people's income is given in the sorted vector p, where an element p_i represents the income of person i, with i running from 1 to N, the total number of people simulated. We call this the 'percentile' (which is also adequate when the number of persons simulated is 100). The production vector w is given by the 'derivative' of the income curve,

$$w_i = \frac{p_{i+1} - p_{i-1}}{2}, \tag{1}$$

with the two special boundary cases given by

$$\begin{aligned} w_1 &= p_2 - p_1 \\ w_N &= p_N - p_{N-1}. \end{aligned} \tag{2}$$

See Figure 1. The total income and production are given by, respectively,

$$P = \sum_{i=1}^{N} p_i,$$

$$W = \sum_{i=1}^{N} w_i. \tag{3}$$

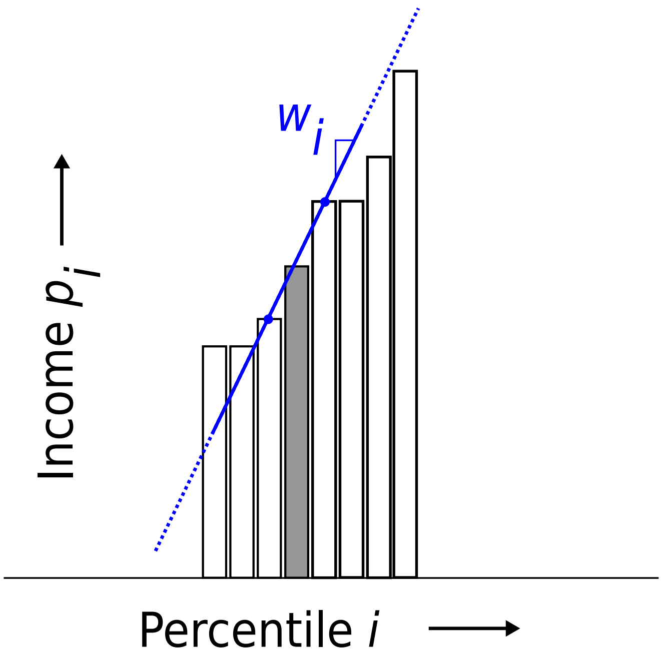

Figure 1: Detail of the distribution of income p_i as a function of percentile i. The production of worker i (shaded) is given by the 'derivative' of the income curve, namely the difference (divided by two) of the incomes of the two immediate neighbors (shown by the blue line), Eq. 1.

We can for instance start with a full-equality distribution. A set of $N = 100$ people that all earn equally ($\forall i : p_i = 1$ unit). The total income is $P = 100$ and the total production is $W = 0$. In the absence of incentives, nobody is doing anything ($w_i = 0$), and people live in misery: The price of things is infinite, price is all income divided by all production, $C \equiv P/W = \infty$. A worker, on average, has a real income of $W/P = 0$. A modal worker (percentile 50) also has a real income equal to his share of income: $c_{50} = W \times (p_{50}/P) = 0$. In fact, any worker has a real income of 0. Society incentivating laziness and dying. "Why should I work? No benefit to be gained from it!"

We can now make a small change to the distribution. We take 1% away from the income of any worker and give it to any other worker. After that we'll sort the array, placing the lowest salary in percentile 1, $p_1 = 0.99$, and the highest salary in percentile 100, $p_{100} = 1.01$. All the others remain unaltered, $p_i = 1$ for $i = 2 \ldots 99$. See Figure 2. We now see that not only the one with higher salary ($i = 100$) starts working, but also worker 1 (who had his salary reduced), as well as workers 2 and 99:

$$\begin{aligned} w_1 &= p_2 - p_1 = 1 - 0.99 = 0.01 \\ w_2 &= (p_3 - p_1)/2 = (1 - 0.99)/2 = 0.005 \end{aligned}$$

$$\begin{aligned} w_{99} &= (p_{100} - p_{98})/2 = (1.01 - 1)/2 = 0.005 \\ w_{100} &= p_{100} - p_{99} = 1.01 - 1 = 0.01. \end{aligned} \tag{4}$$

All others remain inactive. The total production is now given by the sum of the above numbers, $W = 0.03$. The total income remains unaltered at $P = 100$ (since we just transfered salary from one person to another). The price of things has dropped, from infinity to $C = P/W = 100/0.03 = 3300$. A worker, on average, has a real income (in terms of goods that can be bought) of $W/P = 0.0003$, a few crumbs. Even people that do nothing ($i = 3 \ldots 98$) get some crumbs, $c_{50} = W \times (p_{50}/P) = 0.03 \times 1/100 = 0.0003$. The poorest worker gets a little less (99% of that) and the richest a little more (101% of c_{50}). Note that, while we took away income from someone, *everybody* benefits, even that person that was fleeced! And even the 'parasites' – the majority of the population – that do nothing. Moreover, in a system where capital works on a for-profit-basis, and profit is proportional to production – Piketty demonstrates an empirical rule of 5% per year – and the capital is the entity that makes the decisions in production in a free market – a.k.a. 'capitalism' – the system will decide to implement these changes somehow.

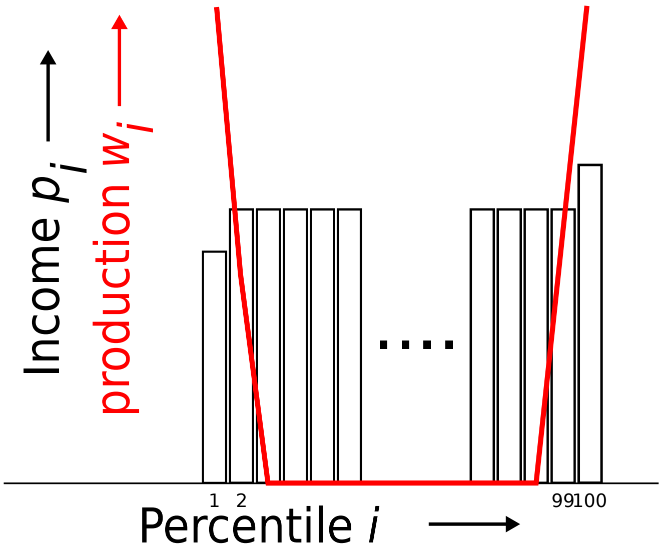

Figure 2: Distribution of income p_i and production w_i as a function of percentile i, after a single step in income redistribution, as explained in the text.

This is not a work on promoting a political dogma, nor will we discuss *how* capitalism manages to implement the changes (for instance through a government that tries to optimize this GDP), we just assume that these changes are implemented. Encouraged by the above results, Let's see how the system will evolve. We now used the following procedure: We take a random person and transferred a random part of his income to a neighbor. We then sort the income vector p and calculate the production of the workers w_i and the total production W. If this production increased, the changes were kept. If not, the previous distribution was kept. Then a new iteration was made. This until the distribution did not change significantly anymore.

The final distribution is given in Figure 3. As we can see it goes completely off-scale. It seems nearly all workers have zero income and zero production, while a small group at the top do all the work and get all the income. The second panel of the figure shows the same in logarithmic scale, to better present the effects.

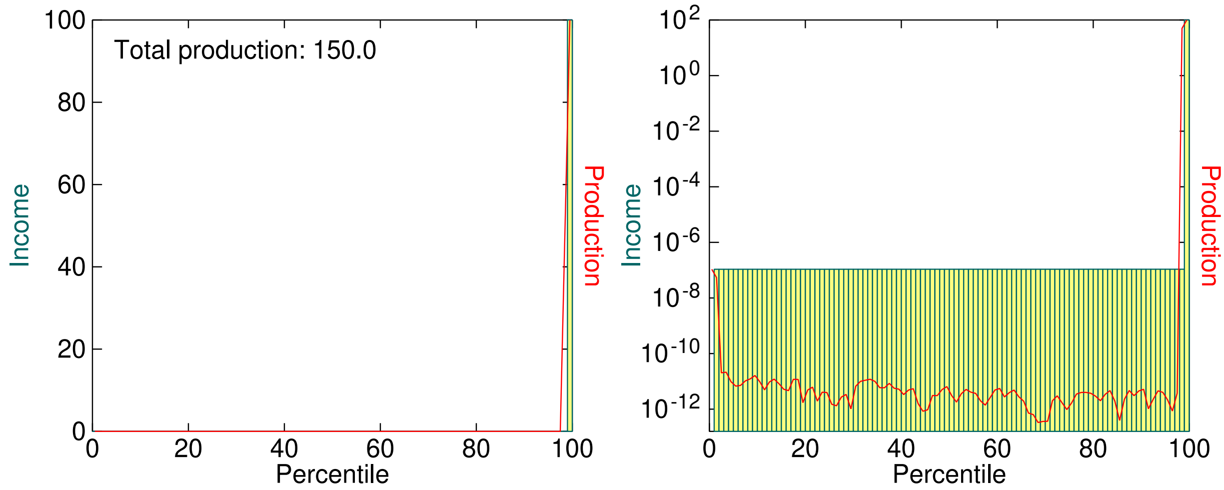

Figure 3: Final distribution of income p_i (rectangles) and production w_i (red line) as a function of percentile i. The left panel shows the results in linear scale, the right panel in logarithmic scale.

First of all, the total production has gone up to $W = 150.0$, with the total income at $P = 100$, the average worker has $W/P = 1.5$ of real income. The poorest worker gets no income whatsoever (it is within the range of significance of the calculations), $p_1 \approx 0$, while he produces $w_1 = 5 \times 10^{-7}$. A modal worker does very little, $w_{50} < 10^{-11}$ and gets a tiny income $p_{50} = 1.08 \times 10^{-7}$. How much can he buy from that? Not much, $c_{50} = 1.6 \times 10^{-5}$, much less than in the situation before. The lion's share of consumption goes to the top percentile, who has an income of $p_{100} = 99.999989$ for which he can buy basically all 150 production. This guy – 'top management' – also has to work a lot, actually doing most of the work, $w_{100} = 99.999989$, but is also flanked by 'middle management' that works a lot, $p_{99} = 49.999995$, but who has nearly no income $p_{99} = 1.08 \times 10^{-7}$, like the modal worker.

3 CONCLUSIONS

The results of our computations can be summarized as:

- There are a few people that work a little, and get no income. The cohort of the 'miserables'

- There are many who do nothing and get a meager salary. The cohort of the 'useless'

- There are few who work tremendously and get the lion's share of income. The cohort of the 'elite'

We can compare this to a stadium where football takes place. One person, let's call him Cristiano Ronaldo, does most of the work and also gets a high reward. He is assisted by some second echelon players to fill the field. They also put in a lot of effort, but get hardly any reward. In the stadium are thousands of spectators that do nothing and get nearly no consumption right. After the match, the janitors do a little work of cleaning up the stadium and preparing the pitch for the next game. They do not get paid.

The question is how realistic is this? Moreover since it only addresses the work market, while there are also other agents in an economy and society. Some doubts can be placed on the validity of the simplistic assumptions:

- Do all workers have all the information about incomes of everybody? Is the market 'efficient'.
- (In capitalism) also capital gets reward. There will be people owning the means of production that will not (have to) work at all and still get rich. They are playing on another board, in another vector field.
- Workers are not only incentivated by their direct neighbors, but also further up and down the income ladder.
- Not all people are equal and equally informed. Some simply have more qualities and produce more with the same incentive
- Not all people are incentivated into work by income (only).
- Production is not a linear function of income difference.
- Income is not something that can be disconnected entirely from production, as was assumed here. In most cases, are incomes a function of productivity.

Yet, we find these results interesting enough and may help in the eternal discussion of wealth (re)distribution.

REFERENCES

[1] `https://probe.org/utilitarianism-the-greatest-good-for-the-greatest-number/`, visited 28 February 2019.

[2] Piketty, T., *Capital in the 21st century.* Éditions du Seuil, Harvard University Press (2014). ISBN 978-0674430006