MATH 64091
Kent State University, Spring 2010

Fake Coins and a Balance Scale
Due Wednesday Jan 27th

In the solutions please avoid using phrases like "in the worst-case scenario". Often, it's hard to tell which case will turn out to be the "worst", so do no neglect even the simplest or most unlikely cases. Write your solutions in complete sentences. Explain all the details.

Example 1. One of the three identically looking coins is fake. It is known that the fake coin is heavier than the other two. How can you decide which one of the three is fake using a balance scale? How many weighings would you need?

Solution. One weighing. Place one coin on one side of the balance scale, one on the other, and put one coin aside. If the scale balances, the coin that we put aside is fake. If one of the sides of the scale is heavier, the fake coin is on that side. □

Example 2. There is a possibility that one of the ten identically looking coins is fake. The fake coin differs in weight from the original ones. How can you decide if there is indeed a fake coin among these 10 coins? How many weighings would you need to determine that?

Solution. One weighing. Put 5 coins on one side of the scale, 5 on the other. If the scale balances, there are no fake coins. If one of the sides is heavier than the other, one of the 10 coins is fake. □

Example 3. One of the nine identically looking coins is fake. It is known that the fake coin is heavier than the other eight. How can you determine, in two weighings, which coin is fake?

Solution. First weighing: three coins aside, three on each side of the scale. This way you will determine three coins which have a fake coin among them. Second weighing: Problem 1. □

Problem 1. One of the 99 identically looking coins is fake. The fake coin differs in weight from the original ones, but it is not known whether the fake coin is lighter or heavier than the rest. How can you determine, in two weighings, if the fake coin is lighter or heavier? What if you had 101 coins?

Problem 2. Seven monkeys and three baby elephants together counterbalance twenty two parrots, while seven baby elephants and and three monkeys counterbalance thirty eight parrots. How many monkeys does a baby elephant weigh?

Problem 3. Given a 3-gallon and a 5-gallon containers, a funnel, and a water tap, pour exactly 4 gallons of water into the 5-gallon container. Would you be able to do this if the containers were of volume 9 and 12 gallons?

Problem 4. Same as Example 3, but it is not known if the fake coin is heavier or lighter than the rest and you are allowed 3 weighings.

Problem 5. Find a set of three weights which you can use to measure any whole number of pounds of sugar from 1 to 10 on a balance scale. Describe how you would measure those amounts of sugar using the weight that you suggest. (You can put the weights on both sides of the scale.)

Problem 6. Two of the six coins are counterfeit and are lighter than the genuine ones. How can you determine, in three weighings, which coins are counterfeit ?

Problem 7. Two of the five coins are counterfeit. One of those two is lighter and the other is heavier than the genuine ones. How can you determine, in three weighings, which coins are counterfeit ?

Problem 8. There are 64 stones of different weights (no two stones weigh the same). Find, in 94 weighings, the heaviest and the lightest of the stones.

Problem 9. You have 10 bags of coins, 10 coins per bag, 10 grams per coin, but in one of the ten bags all the coins are fake each weighing only 9 grams. Use a digital scale only once to find out which bag contains fake coins.

Problem 10. (Bonus) Two out of ten potatoes are radioactive. A Geiger counter can determine if there are radioactive potatoes in a given pile but won't tell how many potatoes are radioactive. Use the counter six times to identify the two radioactive potatoes.

Problem 11. (Bonus) One of the twelve identically looking coins is fake. It is known that the fake coin differs in weight from the other eleven, but you don't know if it is heavier or lighter. How can you determine, in three weighings, which coin is fake? (Same as Example 3 but this time you have 12 coins instead of 9.)

E-mail your questions to soprunova@math.kent.edu. When you are done, you can scan and e-mail your homework to me. You can also fax (330- 672-2209) or mail your homework. Here is my mailing address: Department of Mathematical Sciences Mathematics and Computer Science Building Summit Street, Kent, OH 44242