Telecom Network Systems

Friis Deep Space Com

P. Stallinga

MIEET 4º ano

This text is based on Chapter X, “Information Theory and Physics” of John R. Pierce's “An introduction to Information Theory”.

Nyquist expression for Johnson noise or thermal noise of a hot resistor is given by

$$V_{rms}=\sqrt{4kTRW} \quad . \tag{1}$$

(k is Boltzmann constant, T is absolute temperature, R is resistance, W is frequency bandwidth). If we connect another (cold) resistor to the hot resistor, electric power will flow to this second resistor. Thus, a hot resistor is a potential source of noise power. The maximum possible power supplied (when value of resistance is equal; matched resistances) is then $N = V^2/R$, or

$$N=kTW \quad . \tag{2}$$

With this we can also define a noise temperature of a receiver, namely what temperature of a resistance would add the same noise power as the receiver,

$$T_n=N/kW \quad . \tag{3}$$

Thus, T_n is just a measure of the noisiness of the receiver. As examples: a good radio or television receiver has a noise temperature of about 1500 K. A space-mission receiving station only 20 K.

Remember the channel capacity of Shannon and Hartley

$$C = W \log\left(1 + S/N\right) \quad \text{(bit/s).} \tag{4}$$

(Chapter 0 of lecture notes). This can be written as

$$C = W \log\left(1 + \frac{S}{kTW}\right) \quad \text{(bit/s).} \tag{5}$$

For *W* large and thus *S*/*kTW* small, this can be approximated by (check with Wolfram Alpha)

$$C = \frac{1}{\ln(2)} \frac{S}{kT} \quad . \tag{6}$$

or

$$S = \ln(2) kTC \quad . \tag{6}$$

This says that even if we use a very wide band width, we need at least a power S = 0.693 *kT* joule per second to send one bit per second, so that **on the average we must use an energy of 0.693 *kT* joule for information we transmit**.

Exercise 1) What is the minimum theoretical power consumption of a SATA 3 harddisk with a transfer rate of 6 Gb/s? Are we close to this physical thermodynamical limit, yet?

The approximation of Johnson noise is only valid for low frequencies (approx. 1 THz). Above this frequency the noise is governed by Planck's black body radiation:

$$N = \frac{hfW}{\exp(hf/kT) - 1} \quad . \tag{7}$$

With *h* Planck's constant, and *f* the frequency of radiation. (Check that for $f << kT/h$ this gives Eq. 2).

The most sophisticated communication systems are those used in sending data back from deep-space missions. These are extremely low-noise maser receivers, and they make use of sophisticated error correction, including convolutional coding and decoding using the Viterbi algorithm. In sending pictures from Jupiter and its satellites back to earth, the Voyager spacecraft could transmit 115,200 binary digits per second with an error rate of one in 200 by using only 21.3 watt of power. The power is only 4.4 dB more than the ideal limit using infinite bandwidth.
Pluto is about 5 x 10^{12} meter from earth. Ideally, how fast could we send data back from Pluto? We'll assume noise from space only, and no atmospheric absorption.
If we use a transmitting antenna of effective area A_T and a receiving antenna of effective area A_R, the ratio of received power P_R to transmitted power P_T is given by Friis's transmission formula:

$$\frac{P_R}{P_T} = \frac{A_R A_T}{\lambda^2 L^2} \quad , \tag{8}$$

This is a slightly different format of conventionally encountered in the literature.

Exercise 2) Derive Equation 8.

Exercise 3) Imagine we use radio waves of λ = 1 cm, 10 watt power from Pluto with an emitting antenna of 10 m^2, a receiving antenna of 1000 m^2, and a background temperature of 4 K, what is the maximum bit rate?

Exercise 4) If we switch to visible light waves, λ = 600 nm (red), and use lenses instead of antennas, an emitting 'antenna' area of 1 m^2 and a receiving telescope with 100 m^2 area, what would then be the maximum bit rate? (Assume a background noise radiation temperature of 350 K).

Exercise 5) How many bits per photon is the answer of exercise 4? How can information be passed with more than 1 bit per photon? Describe a way.

Planck Law of black body radiation is

$$I(\nu,T)=\frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1} \quad . \tag{9}$$

Exercise 6) The Sun has a temperature of about 5778 kelvin. At which wavelength it emits more radiation (W λ^{-1} m^{-2}).