

Pulse Propagation in Optical Fibres

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Abstract

This note describes the simulation exercise of the course 34130 Introduction to Optical Communication on group velocity dispersion and self-phase modulation in optical fibres. After introducing a general optical communication system, the propagation of optical pulses in a dispersive fibre will be described with special emphasis on the simple case of a Gaussian pulse, followed by a brief introduction to optical fibre nonlinearities, in particular self-phase modulation. The three exercises, making use of the VPIplayer™ environment, will then be introduced. The main goals of the exercises are: 1) to introduce the relation between the time and frequency domain descriptions of a signal, 2) to observe and calculate the time domain pulse broadening due to dispersion, and 3) to address the power dependence of the fibre behaviour due to the power dependent refractive index.

1 Theoretical background

This exercise deals with transmission impairments in optical fibres due to group velocity dispersion (GVD) and optical fibre nonlinearities. As seen in the lectures, GVD arises from the frequency dependence of the group index of the fibre. This results in a frequency dependent group delay through a given length of optical fibre. Since optical fibres are used to transmit information under the form of light pulses, which have a certain spectral width, the different frequency components of their spectrum will propagate with different group velocities, typically resulting in pulse broadening when observed in the time domain.

The type of nonlinearity that will be observed in this exercise has its physical origin in the small light intensity dependence of the refractive index of the material, usually silica glass, used to make the fibre. This physical effect is called nonlinear Kerr effect and leads, among others, to self-phase modulation (SPM), where the time varying intensity of a light pulse modulates the refractive index of the core of the fibre, which in turns affects the phase of the pulse itself. SPM is responsible for spectral broadening of the information-carrying signal propagating through the fibre.

In this exercise, the manifestations of GVD and SPM upon propagation of a single pulse in an optical fibre will be analysed numerically. In order to remind some of the key relations between the time and frequency domain representations of a signal, a basic study of Gaussian pulses and pulse trains will be conducted first.

1.1 Optical communication systems

A simplified optical communication system is depicted in Fig. 1. Here, the focus is on digital transmission systems where the information is transmitted as a string of bits taking binary values ‘0’ or ‘1’. An electrical binary data stream is used to modulate the intensity of light emitted by

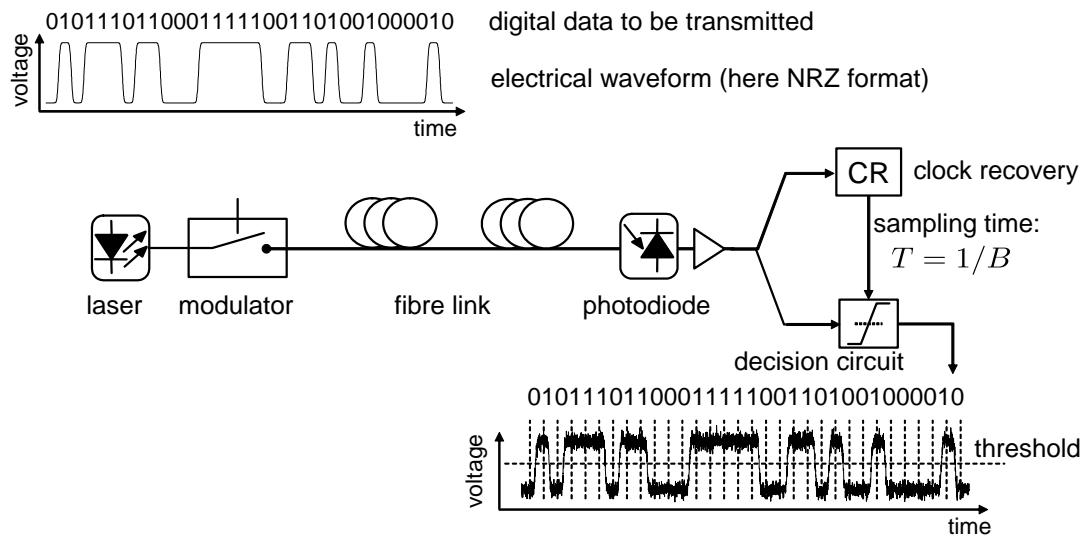


Figure 1 Typical digital optical communication system.

a light source, typically a semiconductor laser. Here, the non-return-to-zero (NRZ) line code, where the intensity of the light remains to its high value whenever a number of consecutive ‘1’s are transmitted, is employed. At the output of the transmitter, one expects to find the optical replica of the electrical information signal. The optical signal then propagates through the optical fibre link. At the receiver side, the optical signal is converted to the electrical domain in a photodiode. A clock recovery circuit is used to extract a clock signal from the detected data. This clock signal determines the instants at which the received signal will be sampled. The recovered samples are then compared to a decision threshold and a decision is made on whether the received bit is a ‘0’ or a ‘1’. Hopefully the recovered data should be identical to the transmitted data. This is not always the case and, in practice, noise and signal distortion can result in bit errors, i.e. a ‘1’ is mistaken as a ‘0’ at the receiver and vice versa. Hopefully proper design of the system prevents this from happening too often (a typical target for optical systems is to have an erroneously detected bit every billion transmitted bits, resulting in a bit-error-ratio (BER) of 1.0×10^{-9}). Coding can, to some extent, be used to detect and correct such errors. The BER of optical communication systems will be the topic of the third simulation exercise in this course.

In the system represented in Fig. 1, the NRZ line coding scheme was used in conjunction with intensity modulation, also known as on-off keying (OOK), since this method is the most commonly used in optical communication systems. Intensity modulation has been the modulation technique of choice in digital optical communication systems since the optical signal can be easily converted to the electrical domain with a photodiode, and the ‘1’ and ‘0’ bits can be easily detected by the presence or absence, respectively, of optical power at the sampling time. However, even for intensity modulation, other choices of the line code are possible. In the case of return-to-zero (RZ) signals, the duration of the optical pulse is shorter than the bit slot (equal to $1/B$, where B is the bit rate, for instance 100 ps at 10 Gbit/s). A consequence is that, as the name suggests, the intensity of the light returns to zero after each transmitted ‘1’, even when several consecutive ‘1’s are transmitted. This essential distinction between NRZ and RZ is illustrated in Fig. 2. The top row shows “ideal” NRZ and RZ optical waveforms¹ corre-

¹What is actually represented is the modulus square of the complex envelope of the signal, which corresponds to the optical power. It should be understood that the optical signal consists of many oscillations of the optical field at a frequency corresponding to $f_0 = c/\lambda_0$, where $c = 299792458 \text{ m} \cdot \text{s}^{-1}$ is the speed of light in vacuum and

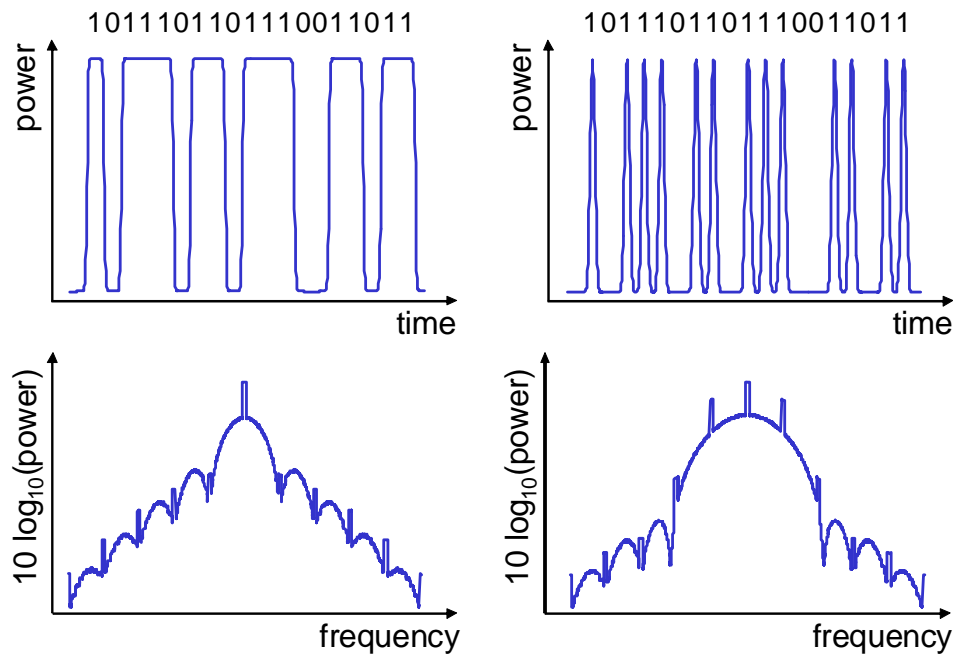


Figure 2 Waveforms and spectra of NRZ (left) and RZ (right) line coded signals. In order to distinguish more easily the shape and main features of the spectra, the discrete values of the simulated power spectrum have been averaged over a bandwidth corresponding to 1/10 of the bit rate.

sponding to the transmitted binary sequence 1011101101110011011. As described previously, the amplitude of the NRZ waveform remains to the “high” level between two consecutive ‘1’s, while that of the RZ waveform returns to the “low” level between consecutive ‘1’s. It should be noted that bandwidth limitations impose a finite rise and fall time on the optical signal.

Under some conditions, the power envelope of RZ pulses could very well be approximated by a Gaussian function. This simplifying assumption will be convenient to study the effect of group velocity dispersion on pulse propagation in what follows and will therefore be retained. In real life, RZ pulses may not have such a Gaussian shape. Qualitatively, this will not change the main conclusions reached in this exercise. However, it would definitely make the analysis more complicated if quantitative results were sought.

1.2 Pulse propagation in a dispersive fibre

1.2.1 General expression of the field

It has been seen in the lectures that each frequency component of the electric field of a lightwave propagating in a single mode optical fibre can be expressed at each point \mathbf{r} according to

$$\tilde{\mathbf{E}}(\mathbf{r}, \omega) = \tilde{B}(0, \omega) F(x, y) e^{j\beta z} \mathbf{x}, \quad (1)$$

where $\tilde{B}(0, \omega)$ is the initial amplitude, $F(x, y)$ represents the field transverse distribution, which is often considered to be Gaussian

$$F(x, y) = \exp\left[-\pi \frac{x^2 + y^2}{A_{\text{eff}}}\right], \quad (2)$$

λ_0 , typically around 1550 nm, is the centre wavelength of the laser. This point will be clarified in Sec. 1.2.1.

β is the propagation constant, x and y are the transverse coordinates of the fiber, z is its longitudinal coordinate, \mathbf{x} is a vector that represents the state of polarisation of the light, and A_{eff} is known as the effective area of the fibre.

The effect of group velocity dispersion arises from the fact that the propagation constant is frequency dependent, $\beta = \beta(\omega)$. Since it is possible to express the electric field in either the time or the frequency domain according to the usual Fourier transforms relations

$$\tilde{\mathbf{E}}(\mathbf{r}, \omega) = \int_{-\infty}^{+\infty} \mathbf{E}(\mathbf{r}, t) e^{j\omega t} dt, \quad (3)$$

and

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{\mathbf{E}}(\mathbf{r}, \omega) e^{-j\omega t} d\omega, \quad (4)$$

the electric field $E(\mathbf{r}, t)$ can be interpreted as an infinite sum of monochromatic waves of frequency ω , each propagating in the fibre with a different propagation constant $\beta(\omega)$. Consequently, for each frequency component

$$\tilde{\mathbf{E}}(x, y, z, \omega) = \tilde{\mathbf{E}}(x, y, z = 0, \omega) e^{j\beta(\omega)z}, \quad (5)$$

hence

$$\frac{\partial \tilde{\mathbf{E}}}{\partial z}(\mathbf{r}, \omega) = j\beta(\omega) \tilde{\mathbf{E}}(\mathbf{r}, \omega). \quad (6)$$

Eq. (6) will be the starting point for the analysis of dispersion-induced pulse broadening in Sec. 1.2.3. It is important to notice at the present time that the present description of propagation in optical fibres is straightforward, *because* the solution (1), known from the lectures, is used as the starting point. A rigorous derivation requires to start all the way from Maxwell's equations, as it has been done in the lectures and in the textbook. Note that in writing (5) a lossless medium has been assumed. The fibre loss can easily be taken into account by multiplying the right-hand term of (5) by the factor $e^{-\frac{\alpha}{2}z}$, where α is the fibre attenuation per unit length (expressed in m^{-1}), and the factor $\frac{1}{2}$ in the argument of the exponential arises from the fact that the propagation of the field – not power – is considered here.

1.2.2 Dispersion parameters

The propagation constant through the fibre, which can be expressed as

$$\beta(\omega) = n(\omega) \frac{\omega}{c}, \quad (7)$$

where n is the mode index, can be expanded around ω_0 according to

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2}\beta_2(\omega - \omega_0)^2 + \frac{1}{6}\beta_3(\omega - \omega_0)^3, \quad (8)$$

where

$$\beta_i = \left(\frac{\partial^i \beta}{\partial \omega^i} \right)_{\omega=\omega_0}. \quad (9)$$

Here, a Taylor expansion up to the third order is estimated sufficient. The coefficients β_1 and β_2 can be related to the mode index according to

$$\beta_1 = \frac{1}{c} \left[n + \omega \frac{dn}{d\omega} \right] = \frac{n_g}{c} = \frac{1}{v_g}, \quad (10)$$

and

$$\beta_2 = \frac{d\beta_1}{d\omega} = \frac{1}{c} \left[2 \frac{dn}{d\omega} + \omega \frac{d^2n}{d\omega^2} \right], \quad (11)$$

where n_g is the group index, v_g is the group velocity and c is the speed of light in vacuum.

For most practical purposes in the study of optical communication systems, it is customary to use the dispersion parameter D to express the dispersion

$$D = \frac{1}{L} \frac{d\tau}{d\lambda} = -\frac{2\pi c}{\lambda^2} \beta_2, \quad (12)$$

where τ is the group delay through a length L of fibre. D is usually expressed in units of ps/(nm·km). Another parameter called the dispersion slope and noted S is often introduced in order to describe the variations of D with respect to wavelength. The dispersion slope, which is defined as

$$S = \frac{dD}{d\lambda}, \quad (13)$$

is usually expressed in units of ps/(nm²·km).

1.2.3 The propagation equation

In this section, the evolution of an optical pulse with distance z into the fibre is calculated. According to (4), it is possible to express the field amplitude in the time domain as a Fourier integral

$$B(z, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{B}(z, \omega) e^{-j\omega t} d\omega. \quad (14)$$

The propagation of the field amplitude in the frequency domain can be easily described from (5)

$$\tilde{B}(z, \omega) = \tilde{B}(0, \omega) e^{j\beta(\omega)z}. \quad (15)$$

It is further assumed that the field amplitude can be decomposed as a slowly varying amplitude, or complex envelope $A(z, t)$, and a fast varying term following the oscillations of the optical field at the carrier frequency ω_0

$$B(z, t) = A(z, t) e^{j(\beta_0 z - \omega_0 t)}. \quad (16)$$

Reporting (15) and the Taylor expansion of the propagation constant (8) into (14), and exploiting the definition (16), leads to the propagation equation for the pulse complex envelope $A(z, t)$

$$\frac{\partial A}{\partial z}(z, t) + \beta_1 \frac{\partial A}{\partial t}(z, t) + j \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial t^2}(z, t) - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial t^3}(z, t) = 0. \quad (17)$$

From (10), β_1 is the inverse of the group velocity v_g . Considering a new system of coordinates moving with the pulse at v_g , i.e. introducing the change of variables

$$Z = z, \quad (18)$$

$$T = t - \beta_1 z, \quad (19)$$

the equation of propagation (17) becomes

$$\frac{\partial A}{\partial Z}(Z, T) + j \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2}(Z, T) - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial T^3}(Z, T) = 0 \quad (20)$$

This is the equation satisfied by the pulse envelope $A(z, t)$ over propagation in a dispersive (and lossless) fibre when its dispersion can be well approximated by the coefficients β_2 and β_3 . However, it is obvious from the derivation of (17) that the propagation equation is much easier to integrate in the frequency domain where it simplifies to

$$\frac{\partial \tilde{A}}{\partial z}(z, \Delta\omega) = j[\beta(\omega) - \beta_0] \tilde{A}(z, \Delta\omega). \quad (21)$$

This is the approach that will be followed in the next section in the case of a Gaussian pulse envelope.

1.2.4 Gaussian Pulse

It has been mentioned in Sec. 1.1 that, under some circumstances, the pulses used in RZ optical communication systems can be approximated by Gaussian functions. For such unchirped Gaussian pulses, the complex envelope² can be expressed as

$$A(0, t) = A_0 \exp\left[-\frac{1}{2} \left(\frac{t}{T_0}\right)^2\right], \quad (22)$$

where A_0 is the peak amplitude and T_0 is the half-width at $1/e$ intensity, which can be related to the more commonly used full-width at half-maximum (FWHM) by

$$T_{\text{FWHM}} = 2\sqrt{\ln 2} T_0. \quad (23)$$

The spectrum of the Gaussian pulse can be calculated by Fourier transformation

$$\tilde{A}(0, \Delta\omega) = \int_{-\infty}^{+\infty} A(0, t) e^{j\Delta\omega t} dt, \quad (24)$$

which is straightforward with the help of well-known relations of the type

$$\int_{-\infty}^{+\infty} e^{-\alpha t^2} e^{j\omega t} dt = \sqrt{\frac{\pi}{\alpha}} e^{-\frac{\omega^2}{4\alpha}}, \quad (25)$$

which basically indicates that the Fourier transform of a Gaussian function is another Gaussian function. It is this property of Gaussian functions that makes Gaussian pulses attractive for the study of dispersion effects in optical fibres. In this case, the pulse envelope $A(z, t)$ at any distance z in the fibre can be calculated analytically, as will be shown in Sec. 1.2.5.

Proceeding with the calculation, the Gaussian pulse spectrum can therefore be shown to be equal to

$$\tilde{A}(0, \omega) = A_0 \sqrt{2\pi T_0^2} \exp\left[-\frac{1}{2} \omega^2 T_0^2\right]. \quad (26)$$

1.2.5 Propagation of a Gaussian pulse in a dispersive fibre

In this section, a method to calculate the evolution of the pulse envelope with distance z into the fibre is presented and applied to the case of a Gaussian pulse. It is assumed that the effect of β_3 can be neglected and therefore only the first three terms in the expansion of the propagation constant (8) are retained. The analysis can be carried out as follows:

²In the present situation of an *unchirped* input pulse, the complex envelope *at the fibre input* is actually a real quantity.

1. First the pulse spectrum at $z = 0$ is calculated using (24).
2. The spectrum is then propagated in the fibre according to (21)

$$\frac{\partial \tilde{A}}{\partial z}(z, \Delta\omega) = j\frac{1}{2}\beta_2(\Delta\omega)^2 \tilde{A}(z, \Delta\omega), \quad (27)$$

hence

$$\tilde{A}(z, \Delta\omega) = \tilde{A}(0, \Delta\omega) e^{\frac{1}{2}j\beta_2(\Delta\omega)^2 z}. \quad (28)$$

From (28), one can observe that the magnitude of the spectrum is not affected by group velocity dispersion.

3. Finally the inverse Fourier transform of the pulse spectrum at z is calculated in order to obtain the temporal envelope

$$A(z, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{A}(z, \Delta\omega) e^{-j\omega t} d\omega. \quad (29)$$

If the three steps above are followed for a Gaussian pulse whose initial pulse shape is given by (22), the pulse envelope $A(z, t)$ can be obtained in a straightforward way as

$$A(z, t) = \frac{A_0 T_0}{(T_0^2 - j\beta_2 z)^{\frac{1}{2}}} \exp\left[-\frac{t^2}{2[T_0^2 - j\beta_2 z]}\right]. \quad (30)$$

Hence the pulse width at distance z in the fibre

$$\frac{T(z)}{T_0} = \sqrt{1 + \left(\frac{z}{L_D}\right)^2}, \quad (31)$$

where a quantity known as the dispersion length

$$L_D = \frac{T_0^2}{|\beta_2|}, \quad (32)$$

has been introduced. Note that $T(z)$ is actually the half width at $1/e$ intensity, which can be related to the full width at half maximum through (23).

1.2.6 Self-phase modulation

Self-phase modulation (SPM) arises from the fact that the refractive index of glass depends on the intensity of the lightwave that propagates through the fibre. The refractive index can be written

$$\bar{n} = n + n_2 \left(\frac{P}{A_{\text{eff}}}\right), \quad (33)$$

where n_2 is called the nonlinear index of refraction. For undoped silica, the value of the nonlinear index is of the order of $n_2 \approx 2.6 \times 10^{-20} \text{ m}^2 \cdot \text{W}^{-1}$. P/A_{eff} , where P is the power of the light, is known as the intensity of the optical field within the fibre. In typical optical communication systems, P is of the order of 1-10 mW. However, this relatively small optical power is confined in a very small fibre core, resulting in effective areas of the order of $80 \mu\text{m}^2$ for standard single mode fibres (SSMF), or even less for other types of fibres. Consequently, the light intensity can

take very large values and the nonlinear term $n_2 P / A_{\text{eff}}$ can no longer be ignored. Considering this effect as a small perturbation, the propagation constant can be written

$$\bar{\beta} = \beta + \gamma P, \quad (34)$$

where the non-linear coefficient

$$\gamma = \frac{2\pi}{\lambda} \left(\frac{n_2}{A_{\text{eff}}} \right), \quad (35)$$

has been introduced. For standard single mode fibres, the value of the nonlinear coefficient is therefore about $1.3 \text{ W} \cdot \text{km}^{-1}$ at 1550 nm .

A modified propagation equation that takes the effects of self-phase modulation and attenuation into account can be easily derived from the analysis of Sec. 1.2.3. The effect of nonlinearity is considered as a small perturbation and (34) is substituted into (21). Writing the resulting equation into the time domain, and adding an extra term to take attenuation into account, the final result is

$$\frac{\partial A}{\partial z} + j \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial t^3} - j \gamma |A|^2 A + \frac{\alpha}{2} A = 0. \quad (36)$$

Eq. (36) is known as the “non-linear Schrödinger equation”. Here, the propagation equation for the pulse envelope has been written in the new system of coordinates moving with the pulse at v_g , while keeping the notations z and t for the space and time variables, respectively, instead of the rather unesthetic Z and T , as it was done previously.

This propagation equation can be easily integrated provided the second and third order dispersion can be neglected (i.e. $\beta_2 = 0$ and $\beta_3 = 0$), and therefore only self-phase modulation and attenuation are taken into account. In this case (36) becomes

$$\frac{\partial A}{\partial z} - j \gamma |A|^2 A + \frac{\alpha}{2} A = 0. \quad (37)$$

This last equation can be easily solved by writing the complex pulse envelope in terms of its modulus and argument

$$A(z, t) = |A(z, t)| \exp[j\phi(z, t)]. \quad (38)$$

Reporting into (37) leads to the following differential equations for the modulus and phase

$$\frac{\partial |A(z, t)|}{\partial z} = -\frac{\alpha}{2} |A(z, t)|, \quad (39)$$

$$\frac{\partial \phi}{\partial z} = \gamma |A(z, t)|^2, \quad (40)$$

hence

$$A(z, t) = |A(0, t)| \exp\left(-\frac{\alpha}{2} z\right) \exp(j\gamma |A(0, t)|^2 z_{\text{eff}}). \quad (41)$$

The phase shift due to SPM (nonlinear phase shift) is therefore

$$\phi_{\text{NL}}(z, t) = \gamma |A(0, t)|^2 z_{\text{eff}}, \quad (42)$$

where the effective length

$$z_{\text{eff}} = \frac{1 - e^{-\alpha z}}{\alpha}, \quad (43)$$

has been introduced. The effective length is used to quantify the length of fibre over which the impact of nonlinearities is significant. Due to fibre loss, the power decays exponentially, and nonlinear effects are therefore stronger at the fibre input. The quantity $|A(0, t)|^2$ corresponds

to the optical power at the fibre input. The self phase modulation induced chirp is therefore equal to

$$\delta\omega(z, t) = -\frac{\partial\phi_{\text{NL}}}{\partial t} = -\frac{\partial}{\partial t} (|A(0, t)|^2) \gamma z_{\text{eff}}. \quad (44)$$

On the leading edge of the pulse the SPM-induced chirp is negative (red-shift) whereas it is positive (blue shift) on the falling edge of the pulse. Therefore self-phase modulation results in the creation of new frequency components over propagation in a fibre, hence in the broadening of the pulse spectrum. It is easy to derive an analytical expression for the SPM-chirp in the case of a Gaussian pulse. One important conclusion of this study of SPM is that, *in the absence of dispersion*, the pulse shape (i.e. the modulus square of the complex envelope $|A(z, t)|^2$, which is the power envelope) is preserved upon propagation in the fibre (apart from fibre loss), while only the phase is affected, leading to spectral broadening in the frequency domain. In case dispersion is also present, the SPM-induced spectral broadening will in turn result to intensity distortion via GVD effects (which strongly depend on the spectral width of the signal).

In the presence of both GVD and SPM, the propagation equation (36) needs to be integrated numerically for a given input pulse envelope $A(0, t)$. This difficulty has constituted one of the main driving forces behind the development of powerful simulation software for optical communication systems³.

2 Part 1: Relation between time and frequency domain descriptions of a signal

In the first exercise, the relation between the time and frequency domain descriptions of a single Gaussian-shaped pulse will be investigated. This exercise will be used to illustrate the relation between the pulse width in the time domain and its spectral width in the frequency domain. The VPIplayer™ environment to be used in the exercise is represented in Fig. 3. An optical Gaussian pulse source, which can generate either a single optical pulse (when the `SignalType` parameter is set to `FixedMarkNumber`), an alternation of ‘0’s and ‘1’s (`SignalType` set to `Alternate`) or a pseudo-random sequence of bits (`SignalType` set to `PRBS_N`), is simultaneously characterised in the time domain and frequency domain by an oscilloscope and an optical spectrum analyser, respectively. The Gaussian pulse FWHM can be set between 10 and 90 ps. In case sequences of pulses are emitted, the bit rate corresponds to 10 Gbit/s, hence each bit slot has a duration of 100 ps.

³Commercial simulation software for optical communication system design include VPItransmissionMaker™ WDM from VPIsystems (<http://www.vpiphotonics.com>), OptSim™ from RSoft (<http://www.rsoftdesign.com>), and OptiSystem from Optiwave (<http://www.optiwave.com>).

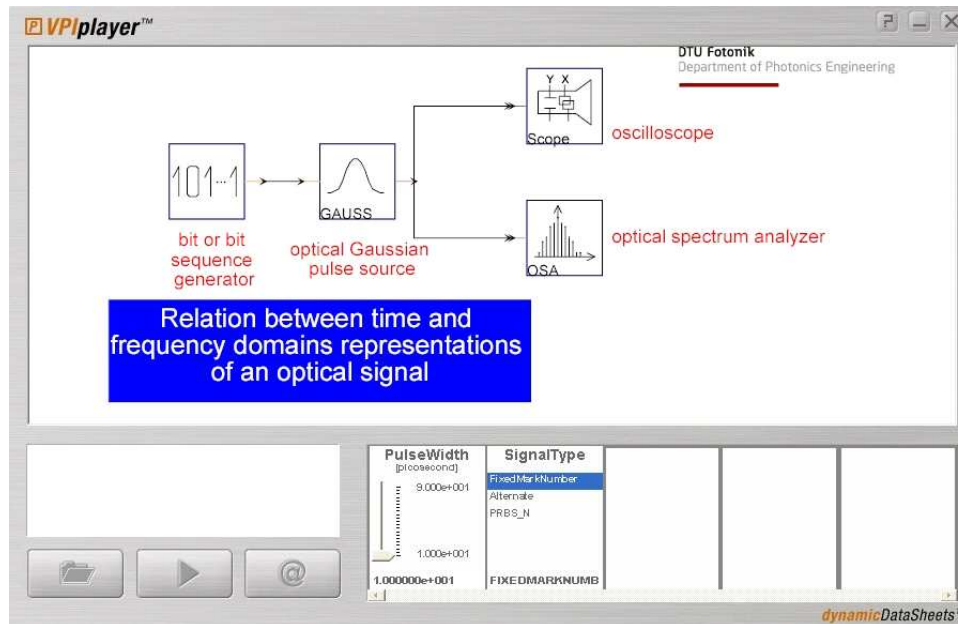


Figure 3 VPIplayer™ environment for the study of the relation between time and frequency domain representations of an optical pulse.

Assignment 1:

1. Open the simulation set-up called Exercise1-1.
2. Set the **SignalType** parameter to **FixedMarkNumber**. In this case a single Gaussian pulse is generated.
3. Change the FWHM pulse width of the Gaussian pulse in the range between 10-90 ps.
4. Using markers, determine the FWHM pulse width T_{FWHM} and the corresponding 3 dB bandwidth of the spectrum ν_{FWHM} .
Make a table with the values of ν_{FWHM} corresponding to the different values of T_{FWHM} .
Hint: The markers can be found by maximising the optical spectrum analyser and oscilloscope windows to full size. Both vertical and horizontal markers can be used. Press on **Create** to create a marker. Two horizontal and two vertical markers can be created, and the difference between their values can be read directly in the display (Δ value).
5. Plot the 3 dB bandwidth ν_{FWHM} as a function of T_{FWHM} .
6. Comment on the relationship between T_{FWHM} and ν_{FWHM} . From the graph, derive a numerical relation linking those two quantities.
7. Make a link with the theory derived in Sec. 1.2.4. From this theory, derive an analytic relation between the Gaussian pulse width T_{FWHM} and its spectral width ν_{FWHM} . At this point it is important to keep in mind that the spectral width that has been retrieved from the simulations corresponds to the 3 dB bandwidth of the *power* spectrum $|\tilde{A}(0, \omega)|^2$. Compare with the relation you have obtained from the numerical simulations in the previous step. Discuss the agreement between the analytical and the numerical analyses.

8. Comment on the shape of the spectra you have observed in the case of a single Gaussian pulse. For a given value of the pulse width, change the **SignalType** to **Alternate**, then to **PRBS_N**.

Observe the differences between the spectrum of a single pulse, that of an alternate ‘...01010101...’ bit pattern and that of a pseudo-random sequence.

Hint: Include time and spectral representations of the signals in your report every time you think it is necessary. This can be easily done by creating screen dumps (using the ‘Print screen’ or ‘PrtSc’ key on your keyboard) when the result window is maximised. The figure can be retrieved (by pasting the content of the clipboard) and processed (for instance cropped) in any picture editing software such as Microsoft Photo Editor or Paint (coming with your Windows distribution), or other excellent - and free - software such as Photofiltre (<http://photofiltre.free.fr>), IrfanView (<http://www.irfanview.com>) and others.

Based on your previous knowledge of Fourier theory, discuss what you observe. Comment on the differences between the spectral properties of the single Gaussian pulse, the alternate sequence and the pseudo-random sequence. Try to relate time domain and frequency domain features of the signal.

3 Part 2: Investigation of dispersion induced pulse broadening

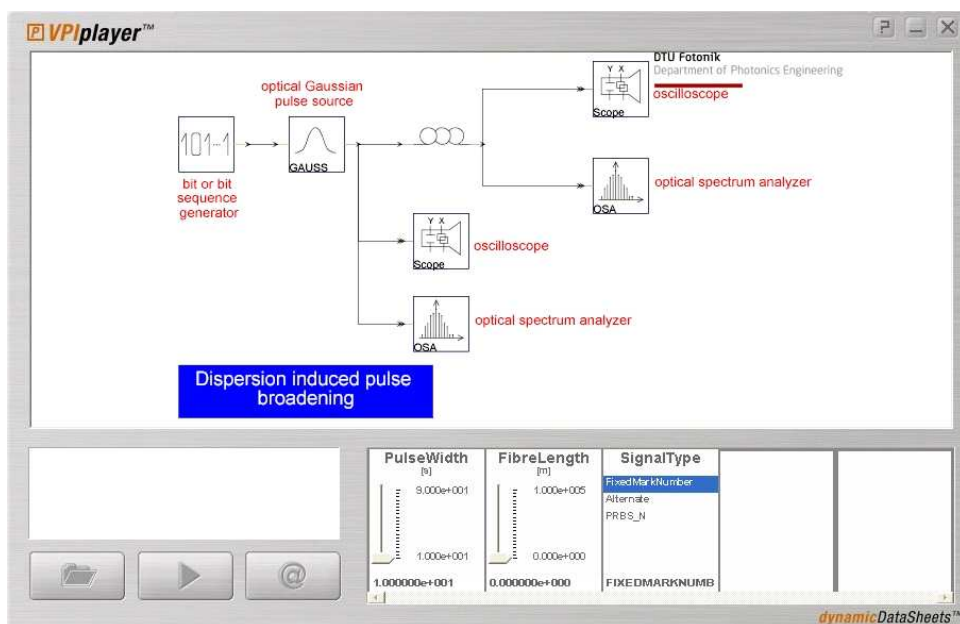


Figure 4 VPIplayer™ environment for the study of dispersion induced pulse broadening in an optical fibre.

In the second exercise, the effect of group velocity dispersion is investigated by propagating a single Gaussian pulse in an optical fibre. The transmission fibre is a standard single mode fibre (SMF) with dispersion parameter $D = 16$ ps/(nm · km) and dispersion slope $S = 0.08$ ps/(nm² · km) at 1550 nm. The simulation set-up is illustrated in Fig. 4. Both the initial pulse width and the length of fibre can be varied using the sliders **PulseWidth** and **FibreLength**, respectively. The type of sequence (i.e. single pulse, alternate or PRBS) can also be changed as in the previous exercise. Here, the focus is on single pulse propagation, but you are also very much welcome to observe what happens with other input signals. Both the waveform and the spectrum can be monitored at the input and output of the fibre. As discussed in the introduction, because of dispersion, the pulse will experience time broadening due to its

non-zero spectral width. In this exercise, the relative time domain pulse broadening will be calculated as a function of the accumulated dispersion.

Assignment 2:

1. Calculate the accumulated dispersion ($D \cdot L$) for the given fibre with fibre lengths of 0, 20, 40, 60, 80 and 100 km.
2. Start the simulation setup called Exercise1-2.
3. Set the `SignalType` parameter to `FixedMarkNumber` – in this case a single Gaussian pulse is generated.
4. Set the pulse width to 30 ps.
5. Set the fibre length to zero first and run the simulation.
6. Determine the pulse width (T_{FWHM}) at the input and output of the fibre.
7. Determine the 3 dB spectral bandwidth (ν_{FWHM}) at the input and output of the fibre.
8. Change the fibre length to 20, 40, 60, 80 and 100 km and repeat the pulse and spectrum characterisations.
9. Change the pulse width to 60 ps and repeat the same procedure as for the 30 ps pulse.
10. Plot T_{FWHM} as a function of accumulated dispersion for the two pulse widths of 30 and 60 ps.
11. Discuss the results, and in particular the relative evolution of the pulse broadening depending on the initial pulse width.
Hint: it could be interesting to normalise the pulse width with respect to the input pulse duration and plot the pulse broadening factor $T(z)/T(0)$ as a function of accumulated dispersion.
Comment on the evolution of the spectral width ν_{FWHM} as a function of accumulated dispersion. Do your observations agree with the theory developed during the lectures and in Sec. 1 of this note?
12. In Sec. 1.2.5, the broadening of a Gaussian pulse propagating in a dispersive fibre has been calculated analytically under the influence of second order dispersion (β_2). In the present case, both β_2 and β_3 are present. However, it is expected that the contribution of β_3 is relatively small compared to that of β_2 , so that it can be ignored.
Compare your numerical results with the analytical expression calculated in Sec. 1.2.5.
Hint: in order to do so, a well chosen normalisation of the horizontal axis for the two curves plotted in step 10 will be useful. After having carried out this normalisation, you are expected to plot the numerical results for the two input pulse widths as well as the analytical results on the *same* graph.
13. Comment on the agreement between the analytical and numerical results.

Comment: in this exercise, you should – hopefully – find an excellent agreement between the numerical and analytical results. Hence you might question the need for performing numerical simulations. As stated above, the analytical solution is quite straightforward *because* a Gaussian pulse has been chosen as the input. Some other pulse shapes might require more sophisticated mathematics in order to derive analytical expressions for the pulse envelope after propagation in the fibre while, *in most cases*, no analytic solution even exists. On the other hand, the

numerical simulations will always enable to characterise the dispersion induced pulse evolution, *for any type of input pulse*. Furthermore, in the realistic case that will be presented in Sec. 4, where both dispersion and nonlinearities are present, it is in general not possible to find an analytic solution for the evolution of the pulse shape in the fibre. In this most general case, numerical simulation is the only feasible approach.

4 Part 3: Investigation of the influence of signal power

In this third and last exercise, the influence of signal power on the propagation of a pulse in a dispersive and nonlinear fibre will be investigated. The goal of this exercise is to observe how high input power will affect the time and frequency domain representations of the signal. The simulation set-up, represented in Fig. 5, is essentially the same as the one of the previous exercise, apart from the fact that the FWHM pulse width is fixed to 50 ps and that the pulse peak power can be adjusted via the parameter `PulsePower`. The dispersion per unit length of the fibre (parameter `Dispersion`) can also be chosen between 0 ps/(nm·km) (that would correspond to a so-called dispersion-shifted fibre, DSF) and 17 ps/(nm·km) (corresponding to standard single-mode fibre, SSMF). The pulse and its spectrum can be visualised at the input and output of the fibre. Be aware that, due to the numerical algorithm used to calculate the propagation of the field in a dispersive and nonlinear fibre⁴, the simulations will be more time consuming than in the previous exercises, especially for high input signal power. The simulation duration will actually increase with increasing signal power.

Assignment 3.1:

1. Open the simulation environment called Exercise1-3.
2. Set the `SignalType` parameter to `FixedMarkNumber` – in this case a single Gaussian pulse is generated.
3. First set the `Dispersion` of the fibre to 0 ps/(nm·km).
4. Set the pulse peak power value (parameter `PulsePower`) to 10 mW.
5. Vary the `FiberLength` and observe the evolution of the pulse width T_{FWHM} and the spectral width ν_{FWHM} . Plot those quantities as a function of fibre length. Does the pulse duration change? What about the spectral width? Is this in agreement with the theory developed in Sec. 1.2.6?
6. Increase the pulse peak power and repeat the step above. What do you observe?

⁴The theoretical analysis of Sec. 1 is actually sufficient to understand intuitively how the calculation is performed. Eq. (28) shows that the effect of dispersion alone can very easily be calculated in the frequency domain, while (41) shows that the effect of self-phase modulation alone is straightforward to calculate in the time domain. The idea behind the numerical algorithm is to divide the fibre length into steps that are sufficiently small so that it can be assumed that the joint effect of dispersion and self phase modulation can actually be calculated independently of each other. Conversion between the time and frequency domains is carried out for each fibre step using a fast Fourier transform algorithm. Since the assumption that dispersion and nonlinearities act independently over a fibre step is dependent on how severe the nonlinear effects are, therefore on the value of the input power, the length of the fibre steps is shortened for high input power, resulting in a sensible increase of computation time.

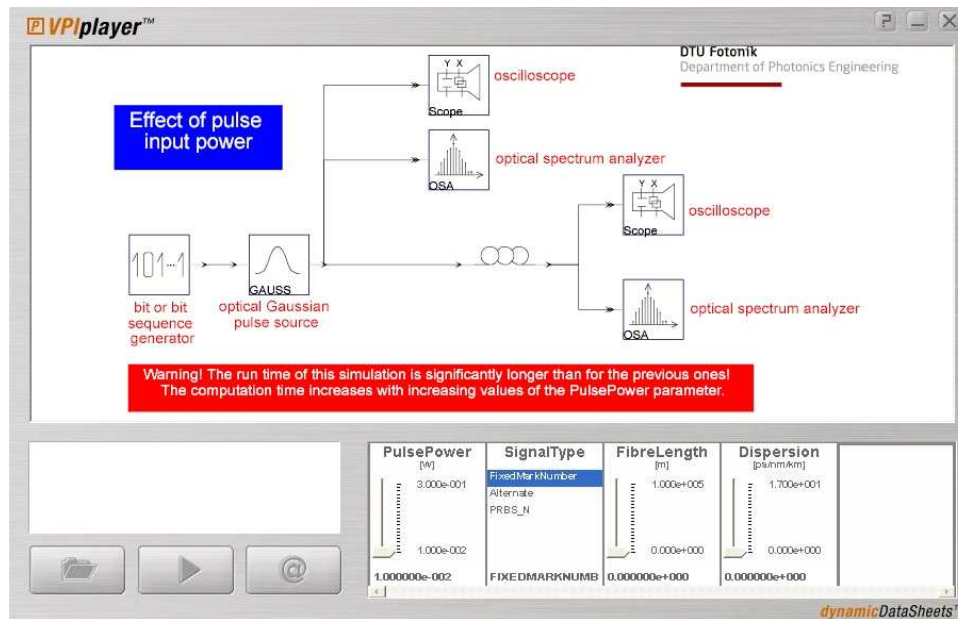


Figure 5 VPIplayer™ environment for the study of the influence of the pulse power on its transmission over optical fibre.

Assignment 3.2:

1. Set the fibre length to 100 km and keep the Dispersion of the fibre to 0 ps/(nm·km).
2. Change the value of the PulsePower parameter in the 10-300 mW range.
3. For each value of the pulse power, observe the pulse waveform and optical spectrum.
4. Plot the 3 dB spectrum width ν_{FWHM} as a function of the input power.
5. Discuss the results.
 What is the general trend of the spectral width evolution with increasing input power?
 In your opinion, is the FWHM (3 dB) spectral width a good figure of merit for describing the spectral evolution of the signal when the input power is increased?
 Describe the changes you observe in the spectrum when the input power is increased.
6. Do you also observe some changes in the temporal representation of the pulse?
7. Still with the fibre length set to 100 km, set the Dispersion to 17 ps/(nm·km).
8. Observe the evolution of the pulse waveform and optical spectrum for different values of pulse power. Compare your observations with the previous situation when the Dispersion was equal to 0 ps/(nm·km).
 Discuss the physical origin of the changes observed in the time domain representation of the pulse.