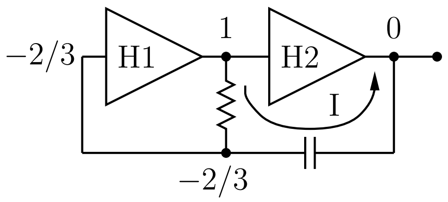
6 This calculation is done for $V_{\mathrm{DD}} = 1$ V. The functionality of the circuit does not depend on the exact value of V_{DD}.
Start with $V_{\mathrm{x}} = 1$ V, $V_{\mathrm{y}} = 0$, $V_{\mathrm{z}} = 1$ V and the capacitor empty, $Q_{\mathrm{C}} = 0$. The resistor feels a voltage drop $V_{\mathrm{x}} - V_{\mathrm{y}} = 1$ V and a current will flow through it. This current can only come from the output of Schmitt Trigger H2 (remember that they are made of opamps) and thus passes through the capacitor, charging it. Initially $I = \Delta V_{\mathrm{R}}/R = 1/R$, but the charging of the capacitor makes it having a voltage drop; the voltage at x is therefore decreasing and with y at a steady 0, the voltage drop across the resistor drops and the current with it. We recognize here a classic relaxation behavior. The current will continue to increase the charge in the capacitor in an ever-decreasing way. This, however, will not continue forever. When the voltage at x drops below 1/3 V the first Schmitt Trigger (H1) will commutate; its output will switch $y : 0 \to 1$. In cascade the second Schmitt Trigger will commutate, $z : 1 \to 0$.
At this moment we start the clock. Just before the commutation, the voltage was 1/3 V at x and 1 V at z; a voltage drop of 2/3 V across the capacitor. Capacitors have the property that voltage drops cannot change instantaneously (because $\Delta V_{\mathrm{C}} = Q/C$ and charge cannot disappear instantaneously; it takes current and time to remove charge). Thus, immediately after the switching of H2, the voltage at x must be $V_{\mathrm{x}} = V_{\mathrm{z}} - 2/3$ V $= -2/3$ V. At the other side of the resistor there is a voltage $V_{\mathrm{y}} = 1$ V and a current will come through the resistor equal to $I = (V_{\mathrm{y}} - V_{\mathrm{x}})/R$ (supplied by Schmitt Trigger H1, passing through C and sinking into H2). The situation at $t = 0$ is as follows:



This current goes through the capacitor and is supplied by the output of H2. Seemingly the current goes against the voltage (from 1 V on the right side to +5/3 V on the left side). This is only seemingly. Don't forget that the current in a capacitor is not proportional to the voltage drop (as a resistor), but proportional to the *time-derivative* of this voltage. The current charges the capacitor in an ever-decreasing way. A classical relaxation behavior with for this case three boundary conditions for the voltage at x:

- Initially, $V_{\mathrm{x}}(0) = -2/3$ V.
- The final voltage, if nothing further happened is, $V_{\mathrm{x}}(\infty) = 1$ V, because at this value the current through the resistor would be zero ($\Delta V_{\mathrm{R}} = V_{\mathrm{x}} - V_{\mathrm{y}}$).
- The relaxation time is $\tau = RC$.

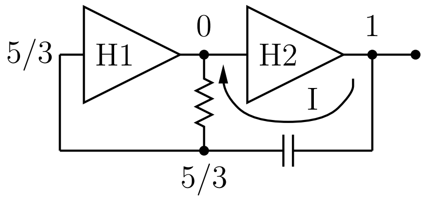
The solution to this exponential decay/approach is

$$V_{\mathrm{x}}(t) = 1 - \frac{5}{3}\exp(-t/RC). \tag{145}$$

When V_{x} reaches two 2/3 V, H1 will commutate. This occurs at a time

$$t_1 = RC\ln(5). \tag{146}$$

At this moment y switches $1 \rightarrow 0$ and in cascade H2 switches, z: $0 \rightarrow 1$. We reset the clock and start observing what happens. Again, the voltage drop in the capacitor is not affected by the switching. Before the switch, the voltage drop was $\Delta V_\mathrm{C} = V_\mathrm{z} - V_\mathrm{x} = 0 - 2/3$ V $= -2/3$ V. After the switch it must be the same, so the voltage at x is $V_\mathrm{x} = V_\mathrm{z} - \Delta V_\mathrm{C} = 1$ V $+2/3$ V $= 5/3$ V. We thus have the following situation at $t = 0$:



A current flows through the resistor and sinks into the output of H1. Thus, the capacitor is discharging. Once again in a relaxation way resulting in an exponential decay/approach. The conditions for the voltage at x are:

- Initially, $V_\mathrm{x}(0) = 5/3$ V.
- The final voltage, if nothing further happened is, $V_\mathrm{x}(\infty) = 0$ V, because at this value the current through the resistor would be zero ($\Delta V_\mathrm{R} = V_\mathrm{x} - V_\mathrm{y}$).
- The relaxation time is $\tau = RC$.

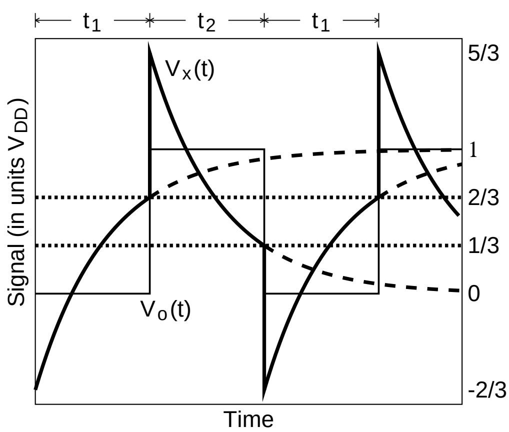
The solution to this exponential decay/approach is

$$V_\mathrm{x}(t) = \frac{5}{3} \exp(-t/RC). \tag{147}$$

It continues to drop until it reaches the commutation voltage of H1, namely 1/3 V. This happens at a time give by

$$t_2 = RC \ln(5). \tag{148}$$

At this moment y switches $0 \rightarrow 1$ and in cascade H2 switches, z: $1 \rightarrow 0$ and we start a new cycle again.
The total period is give by $T = t_1 + t + 2 = 2RC \ln(5)$. To get an oscillation frequency for the square-wave output at 10 kHz we can use values $C = 1$ nF and $R = 31$ kΩ.
Summarizing, the behavior of the circuit is as given below. The solid curve is the actual behavior, the dashed curves are guides-to-the-eye to show where the signal would have gone to. The horizontal dotted lines represent the switching levels, 1/3 V and 2/3 V.



9 a) Define $\beta \equiv R_1/(R_1+R_2)$, $(1-\beta) = R_2/(R_1+R_2)$. Then: $V_\mathrm{p} = \beta V_\mathrm{o} + (1-\beta)V_\mathrm{i}$. In the right commutation point $V_\mathrm{i} = 6$ V, $V_\mathrm{o} = -10$ V, and $V_\mathrm{p} = V_\mathrm{n}$:

$$V_\mathrm{n} = \beta(-10 \text{ V}) + (1-\beta)(6 \text{ V}) = (6-16\beta) \text{ V} \tag{152}$$

In the other commutation point $V_\mathrm{i} = 4$ V, $V_\mathrm{o} = +10$ V, and $V_\mathrm{p} = V_\mathrm{n}$:

$$V_\mathrm{n} = \beta(10 \text{ V}) + (1-\beta)(4 \text{ V}) = (4+6\beta) \text{ V} \tag{153}$$

Two equations with two unknowns. The solution is $\beta = 1/11$ (for example $R_1 = 1$ kΩ and $R_2 = 10$ kΩ) and $V_\mathrm{n} = 50/11 = 4.55$ V (for example $R_3 = 8$ kΩ and $R_4 = 3$ kΩ).
b) The input resistance is defined as $r_\mathrm{i} = \mathrm{d}V_\mathrm{i}/\mathrm{d}I_\mathrm{i}$. The input current is give as $I_\mathrm{i} = (V_\mathrm{i} - V_\mathrm{o})/(R_1+R_2)$. For a constant output voltage, the input resistance is thus equal to R_1+R_2.