Instrumentation 2008-2009 1

1) ⓐ Thermal voltage

$$V_T = \frac{kT}{q} \quad ?$$

$$[V] \stackrel{?}{=} \left[\frac{\frac{J}{K}\cdot K}{C}\right]$$

$$\left[\frac{kg\ m^2}{A s^3}\right] \stackrel{?}{=} \left[\frac{kg\ m^2/s^2}{As}\right] \quad \text{ok!}$$

ⓑ Energy of capacitor

$$E = C^2V/2 \quad ?$$

$$\left[\frac{kg\ m^2}{s^2}\right] = \left[\left(\frac{A^2 s^4}{kg\ m^2}\right)^2\left(\frac{kg\ m^2}{A s^3}\right)\right]$$

$$= \left[\frac{A^3 s^5}{kg\ m^2}\right] \quad \text{no!}$$

$$E = CV^2/2 \quad ?$$

$$\left[\frac{kg\ m^2}{s^2}\right] = \left[\left(\frac{A^2 s^4}{kg\ m^2}\right)\cdot\left(\frac{kg\ m^2}{A s^3}\right)^2\right]$$

$$= \left[\frac{A^2 s^4}{kg\ m^2}\right]\left[\frac{kg^2\ m^4}{A^2\ s^6}\right]$$

$$= \left[\frac{kg\ m^2}{s^2}\right] \quad \text{ok!}$$

S.I.

length	m
mass	kg
time	s
current	A
temperature	K
quantity	mol
luminosity	cd

notes:

- $E = mc^2$ (Einstein)
  $\Rightarrow [J] = [kg\ (\frac{m}{s})^2]$
- $E = qV$
  $\Rightarrow [J] = [A s V]$
  $\Rightarrow [V] = \left[\frac{kg\ m^2/s^2}{A s}\right] = \left[\frac{kg\ m^2}{A s^3}\right]$
- $Q = \int I dt$
  $[C] = [A s]$
- $C = \frac{Q}{V}$
  $[F] = \left[\frac{As}{V}\right] = \left[\frac{A^2 s^4}{kg\ m^2}\right]$

(c) Power of circuit

$$P = I^2/R \quad ?$$

$$\left[\frac{kg\,m^2}{s^3}\right] = \left[\frac{A^2}{\frac{kg\,m^2}{A^2 s^3}}\right] = \left[\frac{A^4 s^3}{kg\,m^2}\right] \quad \text{no!}$$

$$P = I^2 R \quad ?$$

$$\left[\frac{kg\,m^2}{s^3}\right] = [A^2]\cdot\left[\frac{kg\,m^2}{A^2 s^3}\right] \quad \text{ok!}$$

$$P = V^2/R \quad ?$$

$$\left[\frac{kg\,m^2}{s^3}\right] = \left[\left(\frac{kg\,m^2}{A\,s^3}\right)^2 \Big/ \frac{kg\,m^2}{A^2 s^3}\right] = \left[\frac{kg^2\,m^4}{A^2\,s^6}\right]\cdot\left[\frac{A^2 s^3}{kg\,m^2}\right] \quad \text{ok!}$$

notes

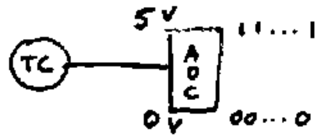
- $R = \frac{V}{I}$ (Ohms Law)

$$\Rightarrow [\Omega] = \left[\frac{kg\,m^2}{A s^3}/A\right] = \left[\frac{kg\,m^2}{A^2 s^3}\right]$$

- $P = E/t$

$$\Rightarrow [W] = [J/s] = \left[\frac{kg\,m^2}{s^3}\right]$$

2) (a)



ADC : 5 Volt in $2^{12}-1$ steps

1.22 mV / step

T.C : $5.88\ \mu V/°C \Rightarrow 1.22\ mV/step$

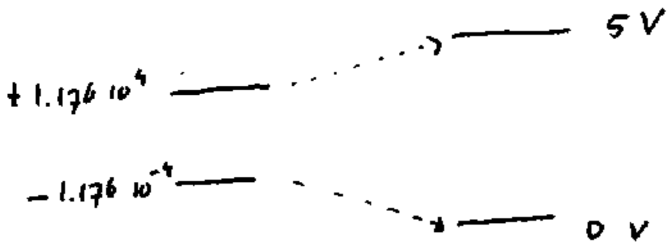
$$= \frac{1.22\ mV}{5.88\ \mu V/°C}\ /step$$

$$= 208\ °C\ /step$$

(b)

$0°C \rightarrow \Delta T = -20°C \Rightarrow V = -1.176\ 10^{-4}\ V$

$40°C \rightarrow \Delta T = +20°C \Rightarrow V = +1.176\ 10^{-4}\ V$



Amplifier : $V_{out} = \left(V_{in} + 1.176\ 10^{-4}\ V\right) \times 21259\ V$

offset gain

→ In this case the resolution of the system will be

$(40°C - 0°C)/(2^{12}-1) = 9.8\ mK$ (0.0098 °C)

or (alternative calculation) : $\frac{208°C}{21259} =$

- what is the energy of a 10 pF capacitor charged to 1 volt?
- check the S.I. units!

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a) Beginning with the definition of a capacitor

$$C \equiv \frac{Q}{V}$$

we see

$$Q = VC \quad , \qquad \text{current is change of charge:}$$

$$I \equiv \frac{dQ}{dt} = C \cdot \frac{dV}{dt} \qquad (C \text{ does not depend on } t!)$$

$$P = V \cdot I = CV\frac{dV}{dt} \quad . \quad \text{Energy is integral of power:}$$

$$E = \int P dt = \int CV \frac{dV}{\cancel{dt}} \cancel{dt} = \frac{1}{2}CV^2 + E_0$$

setting $E = 0$ for $V = 0$:

$$E(V) = \frac{1}{2}CV^2 \quad , \qquad 10\,pF,\ 1V : \quad 5\,pJ$$

b) check units: $\frac{1}{2}CV^2 = [F]\cdot[V]^2$

$$= \left[\frac{C}{V}\cdot V^2\right] = [CV] = [AsV]$$

$$= [Ws] = [J] \quad \text{ok!}$$