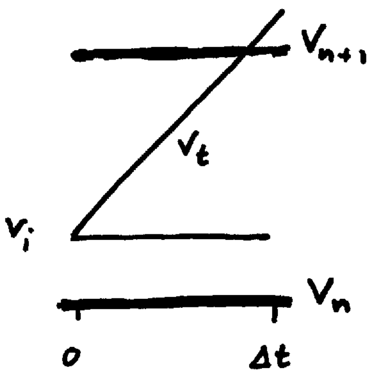
ADC / oversampling

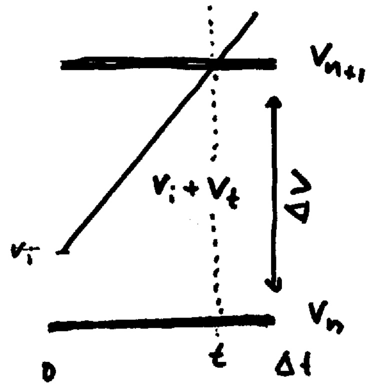


Imagine an infinite number of samples between 0 and Δt. Part of the sum signal $(V_i + V_t)$ will be attributed to V_n and part to V_{n+1}. We want that the average is equal to V_i.

Assume V_t is linear (sawtooth):

$$V_t = V_0 + V_a \times \frac{t}{\Delta t}$$

V_0 = offset
V_a = amplitude



Fraction that returns V_{n+1}:

$$V_i + V_0 + V_a \frac{t}{\Delta t} > V_{n+1}, \text{ with}$$

$$\frac{t}{\Delta t} = \frac{V_{n+1} - V_i - V_0}{V_a}$$

Fraction that returns V_{n+1}: $1 - \frac{t}{\Delta t}$
V_n: $t/\Delta t$

Average: $(1 - \frac{t}{\Delta t}) \times V_{n+1} + \frac{t}{\Delta t} \times V_n = V_i$

Subtract V_n $\left(= (1 - \frac{t}{\Delta t}) V_n + \frac{t}{\Delta t} V_n \right)$

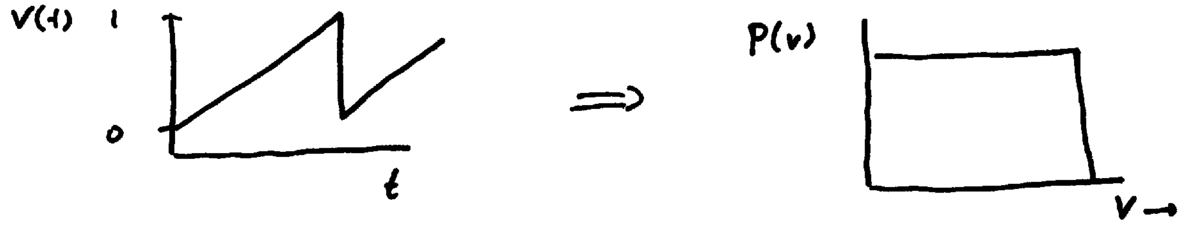
Average: $(1 - \frac{t}{\Delta t}) \times \Delta V = V_i - V_n$

$$\frac{V_a + V_i + V_0 - V_{n+1}}{V_a} \cdot \Delta V = V_i - V_n$$

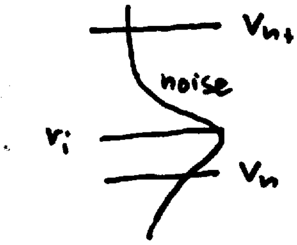
for all values of V_i (!):

$$\frac{V_a + V_0 + (V_i - V_n) - \Delta V}{V_a} \cdot \Delta V = V_i - V_n \Rightarrow \boxed{\begin{matrix} V_a = \Delta V \\ V_0 = 0 \end{matrix}}$$

The above calculation was for a sawtooth function added to the input signal. In other words, a homogeneous probability distribution function.



can other probability functions also be used? For instance white noise, $P(v) = e^{-v^2/\sigma^2}$?



if $V_i + V_{noise} > V_{n+1} \Rightarrow V_{out} = V_{n+1}$

$V_i + V_{noise} < V_{n+1} \Rightarrow V_{out} = V_n$

Average : $P(V_{noise} > V_{n+1} - V_i) \times V_{n+1} + P(V_{noise} < V_{n+1} - V_i) \times V_n$

this should be equal to V_i for all V_i

subtract V_n on both sides

$$P(V_{noise} > V_{n+1} - V_i) \times (V_{n+1} - V_n) = (V_i - V_n)$$

$$P\left(\frac{V_{noise}}{V_{n+1} - V_n} > \frac{V_{n+1} - V_i}{V_{n+1} - V_n}\right) = \frac{V_i - V_n}{V_{n+1} - V_n}$$

In other words, the probability of the (scaled) noise being larger than the (scaled) distance of the input signal to the upper ADC level, V_{n+1}, is proportional to this distance. The only probability function that has this property is the homogeneous distribution of the sawtooth "noise".