

FCE

Problem sheet 1

The best

What is the material used for most of today's semiconductor devices and why. What are the limitations of this material?
Try to express it in numbers, for instance so-called Figures of Merit.

FCE

Problem sheet 2

Case Study

Attached is an article of Stibal *et al.*, “Contactless Electron Mobility Evaluation of Semi-Insulating GaAs and InP Wafers” which proposes a new measurement technique for determining the mobility. We will only analyze the part with the new conductivity measurement technique COREMA.

- 1) Derive Equation (1) on page 2. Remember that the capacitance C of two parallel metal plates is given by

$$C = \epsilon_r \epsilon_0 A/d$$

with ϵ_0 the permittivity of vacuum (see appendix E of sebenta), ϵ_r the relative permittivity ($\epsilon_r = 1$ for vacuum, and close to 1 for air. In the article it is simply called ϵ), A the area of a metal plate and d the distance between them.

- 2) What kind of signal amplitude can we expect in our charge transients for a silicon wafer with resistivity 1 Ωcm , thickness 2 mm and the measuring electrode with diameter 2.5 mm and kept at a distance of 0.1 mm from the wafer after a voltage step $V = 0 \text{ V}$ to 1 V ?

- 3) Explain what the advantages and disadvantages are of using the COREMA technique.

Contactless Electron Mobility Evaluation of Semi-Insulating GaAs and InP Wafers

Rudolf Stibal, Ulrich Kretzer* and Wolfgang Jantz

Fraunhofer Institut Angewandte Festkörperphysik, 79108 Freiburg, Germany

email Wolfgang.Jantz@iaf.fraunhofer.de, phone 49 761 5159510

* Freiberger Compound Materials, 09599 Freiberg, Germany

email kretzer@fcm-germany.com, phone 49 3731 280176

Abstract

An innovative procedure to evaluate the electron mobility of semi-insulating GaAs and InP wafers is presented. It is based on a capacitive contactless technique previously developed for resistivity mapping (COREMA). The measurement is consecutively performed with and without a magnetic field. The correlation with Hall data is satisfactory, whereas the agreement with calculated drift mobility is excellent. The new technique is superior to the conventional procedure in regard of cost, speed, nondestructiveness, reproducibility and lateral resolution. A system design meeting the requirements of eventual routine industrial application is described.

INTRODUCTION

Semi-insulating GaAs and InP wafers, used to fabricate microwave and high speed digital circuits, are routinely characterized with respect to the electrical resistivity ρ and the carrier mobility μ . As a rule the vendor uses test wafers taken from the seed and the tail of the single crystal ingots to obtain the absolute values and variation ranges. The established Hall evaluation technique requires to cut square samples out of test wafers, then to form Ohmic contacts and bond wire or needle contacts. These preparations and the subsequent measurements are time-consuming and destructive, allowing to obtain only some few localized data from sacrificed test material.

In recent years an innovative contactless resistivity mapping technique (COREMA, originally referred to as TDCM), using a capacitive transient charge measurement, has been developed [1]. This technique, briefly described below, is presently used by substrate fabrication companies for routine material quality assurance and is supporting exploratory material research in academic institutes [2-4].

However, a significant drawback of standard COREMA has been that it does not allow to measure μ . Hence, while substantial improvement, acceleration and cost reduction of the

ρ evaluation was provided, the Hall measurement was not entirely substituted and, therefore, still had to be done in addition to the COREMA evaluation. We report on a decisive upgrading of COREMA [5] which allows to evaluate both ρ and μ , thus providing the full set of electric transport parameters. Comparative measurements will be presented for GaAs only, but the method is likewise applicable to semi-insulating InP.

EXPERIMENTAL PROCEDURE

The basic COREMA procedure [1] is briefly reviewed here. It evaluates the time dependent electric charge distribution on a capacitive probe after application of a voltage step. The probe, shown schematically in Fig. 1, consists of a cylindrical metal stub, surrounded by a guard electrode, and a metal chuck that provides both the electrical back contact and mechanical support of the wafer to be measured. The stub is approached to the wafer surface until the distance equals about 1/10 of the thickness of the wafer. A precise horizontal xy stage serves to translate the wafer underneath the probe in order to generate a resistivity topogram.

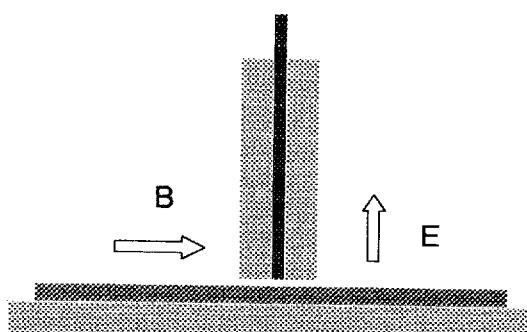


Fig 1. Schematic representation of the capacitive probe

The equivalent circuit of the arrangement, as shown in the insert of Fig. 2, consists of an air capacitor and a sample capacitor that contains the portion of the wafer material below the stub as a lossy dielectric medium. The finite

resistivity of the wafer material accounts for the shunt resistor.

The exponential time dependence of the charge $Q(t)$, as shown in Fig. 2, allows to measure the relaxation constant τ and the charges $Q(0)$, $Q(\infty)$. Using these quantities and the dielectric constant ϵ , the resistivity is calculated according to

$$\rho = \tau Q(0) / \epsilon_0 \epsilon Q(\infty) \quad (1)$$

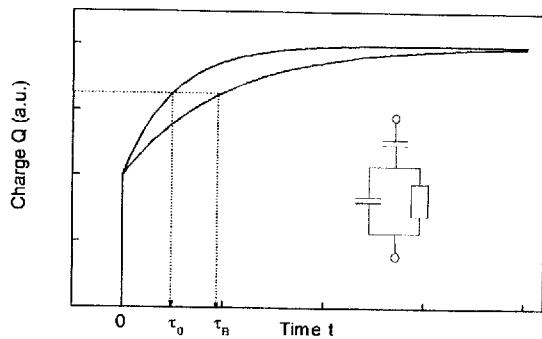


Fig. 2 Charge transients observed after application of a voltage step. The influence of the magnetic field B is indicated. The insert shows the equivalent circuit of the capacitive probe.

The capacitive evaluation avoids the necessity to cut samples from wafers and to prepare ohmic contacts. It is very fast (100 ms per data point), highly repeatable (better than 1 %, limited by sample temperature variations) and yields topographic ρ images of whole wafers with high lateral resolution ($\leq 1\text{mm}$). For these reasons, COREMA is considered superior to conventional contacting techniques, e.g. van der Pauw or linear four point probing.

The contactless measurement of the electron mobility μ relies on the magnetoconductance effect. Conventional Drude theory predicts that the conductance $\sigma(0)$ measured without magnetic field is modified by the application of a finite magnetic field B according to

$$\sigma(B) = \sigma(0) [1 + (\mu B)^2]^{-1} \quad (2)$$

The relative orientations of \mathbf{B} and the electric field \mathbf{E} in the sample capacitor are indicated in Fig. 1. The charge transfer transients, observed with and without applied magnetic field \mathbf{B} are shown schematically in Fig. 2, yielding respectively the time constants $\tau(0)$ and $\tau(B)$. By repeating the measurement at different magnetic fields, the data shown in Fig. 3 are obtained. The expected quadratic dependence is clearly indicated; exhibiting a standard deviation below 0.5% with respect to a quadratic fit. Hence, referring to eq. 1, the resistivity that determines the discharge of the sample capacitor must also depend quadratically on \mathbf{B} , implying

$$\rho(\mathbf{B}) = \rho(0) [1 + (\mu B)^2] \quad (3)$$

Using eqs. 1 and 3, the mobility is evaluated according to

$$\mu = B^{-1} ([\tau(\mathbf{B})/\tau(0)] - 1)^{1/2} \quad (4)$$

Comparative Hall and magnetoconductive measurements were done with samples cut from different ingots, covering a wide mobility range. The Hall data were obtained by the

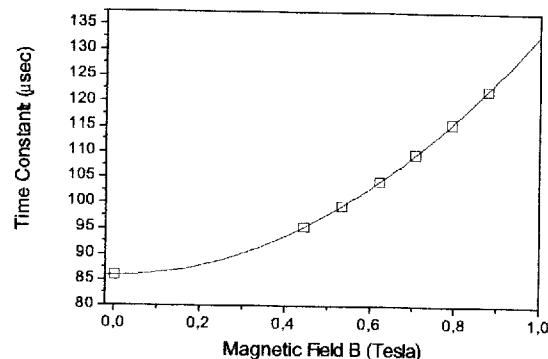


Fig. 3 Dependence of the charge transient time constant on the magnetic field, oriented horizontally, perpendicular to the electric field in the sample capacitor.

standard van der Pauw technique with 27mmx27mm samples, using a magnetic field $B = 0.4357\text{ T}$. The magnetoconductive data were obtained at $B = 0.843\text{ T}$ with a $2.5\text{ mm}\varnothing$ capacitive probe, positioned in the center of the samples. The reproducibility of these measurements, with the samples removed and reinserted, is about 1%. The data are summarized and compared in Fig. 4.

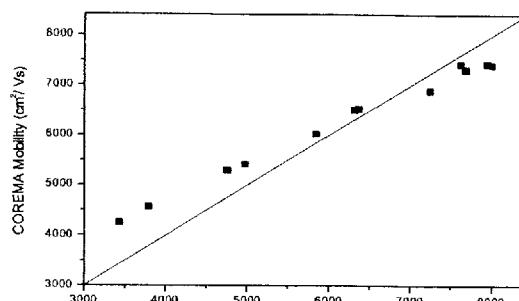


Fig. 4 Correlation of mobility data obtained by Hall and magnetoconductive measurement.

DISCUSSION

Two conclusions can immediately be drawn from the data in Fig. 4. First, a very satisfactory agreement of the absolute mobility values is obtained, giving confidence that magnetoconductance, as measured with the capacitive technique, is a reliable tool to determine mobility.

On the other hand, a systematic discrepancy appears to exist, such that, for low mobility, the magnetoconductive method yields about 20% higher values as compared to the Hall data. The difference decreases with increasing mobility and at high mobility the magnetoconductive data even appear to fall slightly below the Hall data.

These discrepancies may be due to a number of more or less significant reasons. For instance, as stated above, the measurements were done at significantly different magnetic fields. In addition, systematic differences of the mobility values as determined by the two measurement procedures are indeed expected and have been analysed theoretically [6]. Of course, such eventual systematic differences would not compromise the innovative approach, but would be included into the magnetoconductive evaluation routine, if Hall equivalent data were desired.

Both sets of data can be compared to mobility values derived from ionized impurity scattering theory. For most samples the shallow donor concentration is small and the net acceptor concentration is known. Because the net acceptors are compensated by the mid-gap donor EL2, the concentration of ionized impurities is about twice the net acceptor concentration. Application of Brooks-Herring formalism [7] allows to calculate the mobility reduction, compared to impurity-free material, as a function of acceptor concentration. For the lattice-limited mobility a value of $8000 \text{ cm}^2/\text{Vs}$ was assumed. As shown in Fig. 5, the agreement with COREMA data is excellent.

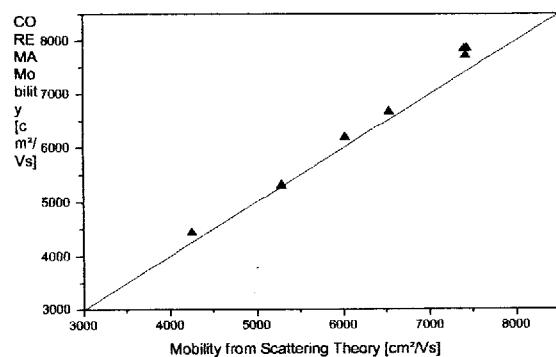


Fig.5 Comparison of mobility data obtained by magnetoconductive measurement with drift mobility obtained from ionized impurity scattering

In the resistivity range above $3 \times 10^8 \Omega\text{cm}$ increasing inhomogeneity of carrier concentration may cause a reduction of Hall mobility, whereas drift mobility is not influenced by inhomogeneities. A similar phenomenon is described in [8] for the resistivity range $10^3 \Omega\text{cm}$ to $10^7 \Omega\text{cm}$. The COREMA measurement appears to be less influenced by carrier inhomogeneities than the Hall method.

A detailed investigation and discussion of these aspects is presently pursued, but is beyond the scope of this paper.

OUTLOOK

It is obvious that the contactless, COREMA-based mobility measurement offers a number of significant benefits, because the advantages of the capacitive resistivity measurement are essentially made available for the mobility evaluation. Hence, the procedure is fast, in particular because no sample preparation is required. Standard production wafers can be measured nondestructively, allowing to perform a reliable screening with any desired number of wafers per ingot without loss of material. While high resolution μ topograms may not be needed, an assessment of the mobility homogeneity across the wafer area, based on a customer-defined measurement plan, may easily be realized. Note, however, that the lowest mobility that can be evaluated is on the order of $1000 \text{ cm}^2/\text{Vs}$.

The data reported in Fig.4 have been obtained with a laboratory setup using a DC current magnet system. Such an apparatus would be disadvantageous for routine measurements in an industrial environment in regard of cost, volume and power consumption. To supply a strong magnetic field in the plane of large diameter wafers would require a very voluminous system and considerable magnetic field buildup

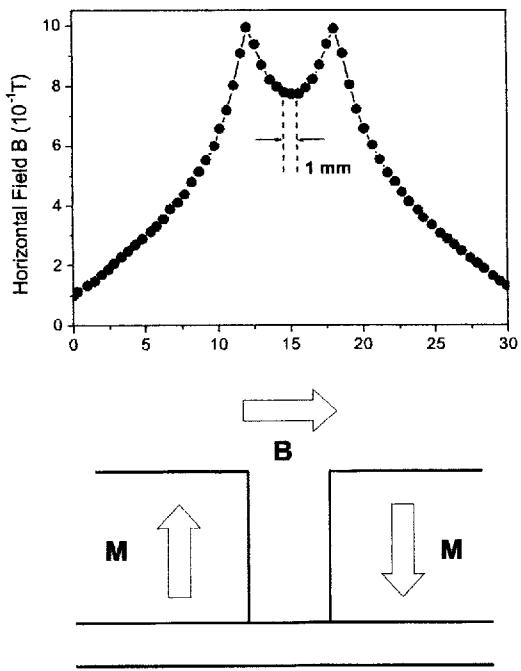


Fig. 6 Schematic representation of the permanent magnets supplying the horizontal magnetic field B . The upper part shows the calculated horizontal magnetic field in the plane 0.5 mm above the permanent magnets (data courtesy Vacuumschmelze).

time would elapse for each data point, because the mobility evaluation requires consecutive measurements with and without magnetic field.

We have, therefore, designed and tested a field generation arrangement with NdFeB permanent magnets, as indicated schematically in Fig. 6, lower part. Note that a strong horizontal magnetic field is required only within the measured material volume, which is located directly on top of the permanent magnets and is determined by the capacitive probe and the thickness of the wafer.

Fig. 6, upper part shows that at this location the horizontal field component exceeds 0.8 T. Within a disk with 1 mm \varnothing and 0.5 mm thickness it is homogeneous to within a few %. Because it nevertheless declines rapidly in the vertical direction, the magnetic field is effectively removed from the measured volume by lowering the permanent magnet arrangement relative to the sample by about 30 mm. This allows to measure $\tau(B)$ and $\tau(0)$ consecutively in a very convenient way.

CONCLUSION

A superior and cost-saving technique to evaluate the mobility of semi-insulating GaAs and InP wafers has been demonstrated. Excellent reproducibility and satisfactory correlation with conventional Hall data has been obtained. The agreement with ionized impurity scattering theory is most convincing. An application oriented system design using an arrangement of permanent magnets is described.

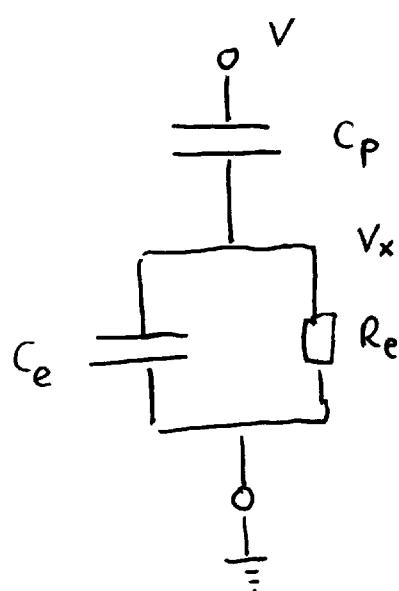
REFERENCES

- [1] R. Stibal, J. Windscheif, W. Jantz, *Semicond. Sci. Technol.* 6 (1991) 995.
- [2.] S.M. Young, M.R. Brozel, C.J.L. Moore, *Mat. Sci. Eng. B44* (1997) 203.
- [3.] M. Wickert, R. Stibal, P. Hiesinger, W. Jantz, J. Wagner, M. Jurisch, U. Kretzer, B. Weinert, Proc. SIMC-X (IEEE publishers 1999) 21.
- [4] M.Fukuzawa, M.Yoshida, M. Yamada, Y. Hanaue, K. Kinoshita, Proceedings of DRIP-IX, Rimini, 2001 (tbp)
- [5] R. Stibal, P.Hiesinger, M. Wickert, W. Jantz, DGKK Arbeitskreis (March 2001), unpublished.
- [6] see e.g. N. Braslau, *Inst. Phys. Conf Ser* 74 (1984) 269 and references given there.
- [7] D. C. Look, *Electrical Characterization of AaAs Materials and Fdevices*, Wiley& Sons (1989, reprinted 1992) /78
- [8] W. Siegel, J. Monecke, S. Schulte, G. Kühnel, *J. Appl. Phys.* 81 (1997) 3155

ACRONYMS

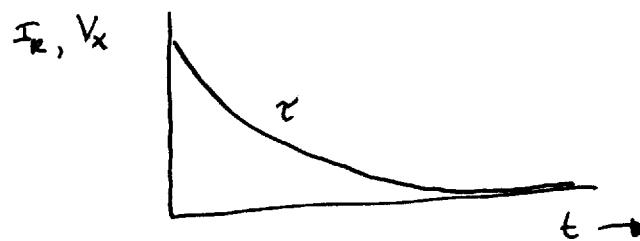
COREMA	Contactless Resistivity Mapping
TDCM	Time Domain Charge Measurement

Solution Problem Sheet 2

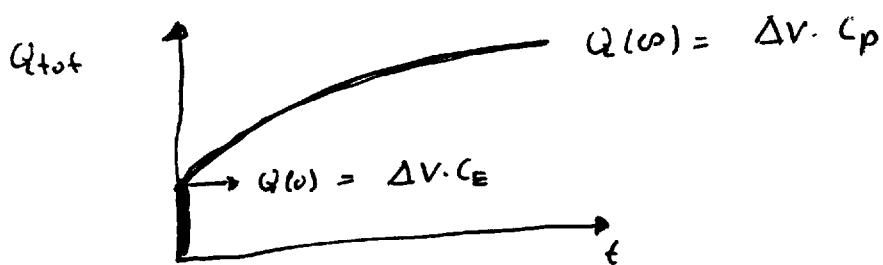
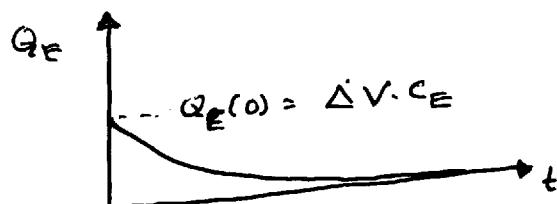
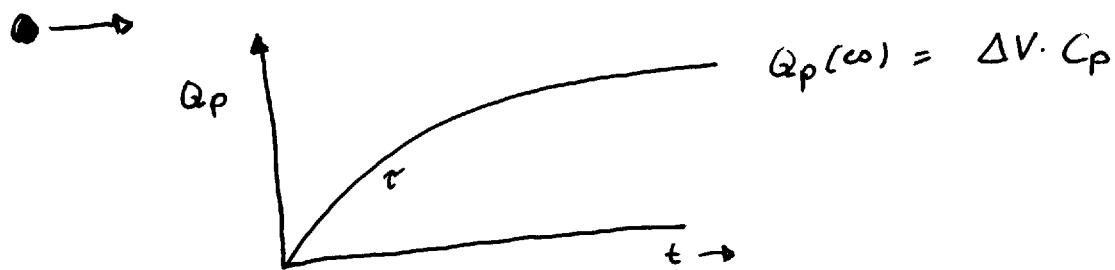


After a pulse $0 \rightarrow \Delta V$

Remember from Electronics:
 $t = \infty ; I_R = 0$

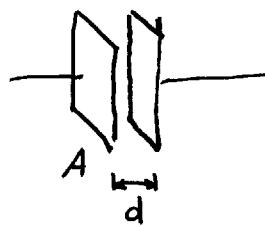


$$\tau = R_e \cdot (C_e + C_p)$$

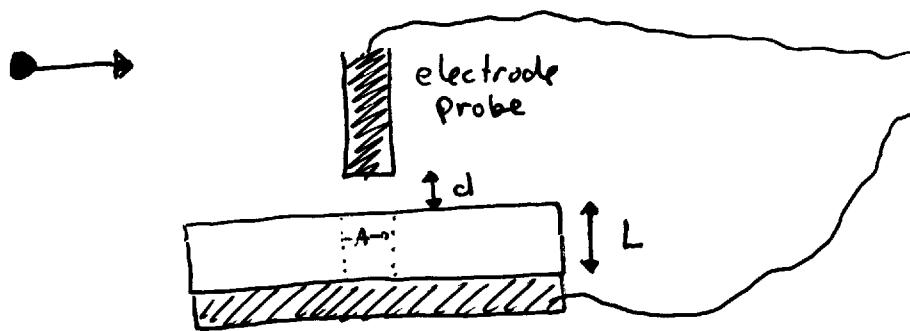


With this it is possible to calculate the equation (1) on p 2 :

→ A capacitor of parallel metal plates :



$$C = \frac{\epsilon_0 \cdot \epsilon_r \cdot A}{d}$$



$$C_p = \frac{\epsilon_0 \cdot \epsilon_r \cdot A}{d} = \frac{\epsilon_0 A}{d} \quad (\epsilon_r \text{ air or } \epsilon_r \text{ vacuum} = 1)$$

$$C_e = \frac{\epsilon_0 \epsilon_r \cdot A}{L} \quad (\epsilon_r \text{ material})$$

$$R_p = \rho \cdot \frac{L}{A}$$

$$C_p = \frac{\epsilon_0 A}{L} \cdot \frac{L}{d}$$

$$\rightarrow \tau = R_p \cdot (C_e + C_p) = \rho \cdot \frac{L}{A} \left(\frac{\epsilon_0 \epsilon_r \cdot A}{L} + \frac{\epsilon_0}{L} \cdot A \cdot \frac{L}{d} \right)$$

$$= \rho \epsilon_0 \left(\frac{L}{d} + \epsilon_r \right)$$

$$\rightarrow \frac{\tau Q(0)}{\epsilon_0 \epsilon_r Q(\infty)} = \rho \epsilon_0 \left(\frac{L}{d} + \epsilon_r \right) \cdot \frac{1}{\epsilon_0 \epsilon_r} \cdot \frac{\Delta V \cdot \epsilon_0 \epsilon_r \frac{A}{L}}{\Delta V \cdot \epsilon_0 \frac{A}{L} \cdot \frac{L}{d}}$$

$$= \rho \cdot \left(\frac{L/d + \epsilon_r}{L/d} \right) \approx \rho \quad (d \ll 0.1 L) \quad (2)$$

$$2) \quad \epsilon_0 = 8.85418 \cdot 10^{-12} \text{ F/cm}$$

$$= 8.85418 \cdot 10^{-12} \text{ F/m}$$

$$A = \pi \left(\frac{d}{2}\right)^2 = 4.9 \cdot 10^{-6} \text{ m}^2$$

$$\epsilon_{si} = 11.9$$

$$L = 2 \text{ mm} = 2 \cdot 10^{-3} \text{ m}$$

$$d = 0.1 \text{ mm} = 10^{-4} \text{ m}$$

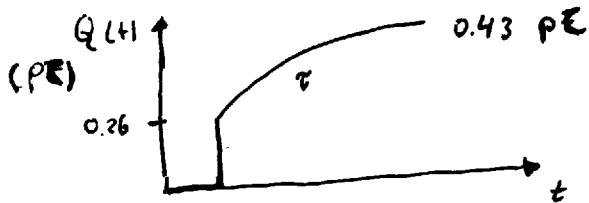
$$\Delta V = 1 \text{ V}$$

$$C_p = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{8.85418 \cdot 10^{-12} \cdot 1 \cdot 4.9 \cdot 10^{-6}}{10^{-4}} \left(\frac{\text{F}}{\text{m}} \cdot \frac{\text{m}^2}{\text{m}} \right)$$

$$= 4.338 \cdot 10^{-13} \text{ F}$$

$$C_E = \frac{\epsilon_0 \epsilon_r A}{L} = \frac{8.85418 \cdot 10^{-12} \cdot 11.9 \cdot 4.9 \cdot 10^{-6}}{2 \cdot 10^{-3}}$$

$$= 2.581 \cdot 10^{-13} \text{ F}$$



$$\tau = \epsilon \epsilon_0 \left(\frac{L}{d} + \epsilon_r \right)$$

$$= 0.01 \text{ } \mu\text{m} \cdot 8.85418 \cdot 10^{-12} \frac{\text{F}}{\text{m}} (20 + 11.9)$$

$$= 2.82 \text{ pF } \mu\text{m} = 2.82 \text{ ps}$$

$$\text{note } [F \mu\text{m}] = \frac{C}{V} \cdot \frac{V}{A} = \frac{C}{A} = \frac{C}{C/\text{s}} = \text{s}$$

- 3) - The amplitudes are small ; ρ^c
The time scale is short ps
 \hookrightarrow difficult to measure
- Thick samples or impractical close proximity of electrode to satisfy condition $d \ll L$
 - + Fast scanning
 - + contact less
 - + mapping of surface

FCE

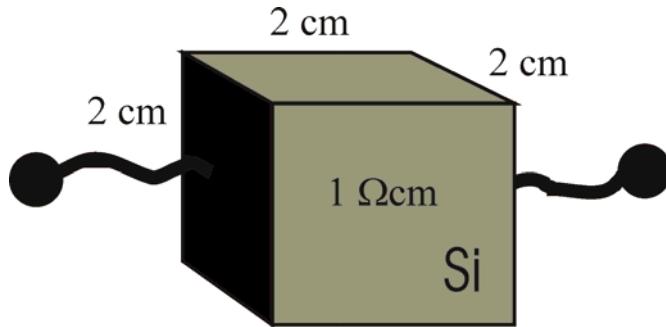
Problem sheet 3

Drude model

1) On basis of the Drude model, which metal should have the highest conductivity:

- a) Gold (Au)
- b) Aluminum (Al)
- c) Copper (Cu)
- d) Silicon (Si)

Is this in line with the observation?



2) A cube (dimensions 2 cm x 2 cm x 2 cm), made of silicon with resistivity $\sigma = 1 \Omega\text{cm}$ is connected to a 1 V source. What are the values of

- a) Resistance, R .
- b) Current, I , and current density, j .
- c) Average speed, v , of electrons.
- d) Electron density, n .

What will be the current if both sides of the cube are covered with 1 μm of SiO_2 ?

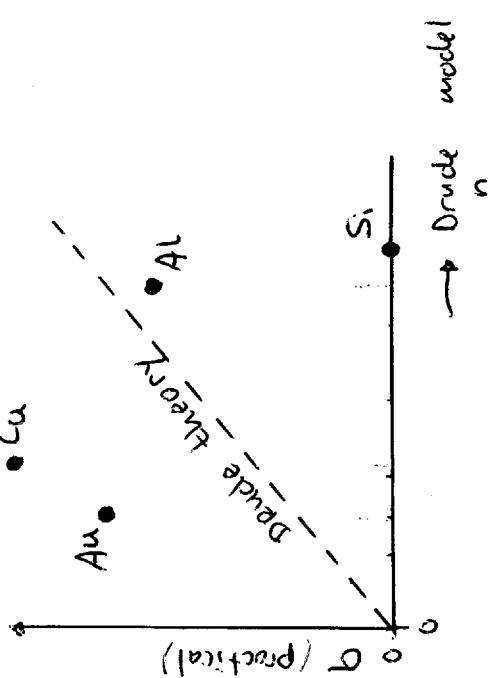
Solution Problem Sheet 3 Drude Model

	① config.	② valence	③ density	④ atomic weight	⑤ electron density / n (cm ⁻³)	⑥ resistivity ρ (Ω cm)
	v	d (kg/m ³)	w (g/mol)			
Au	5d ¹⁰ 6s	1	19300	196.967	5.9 10 ²²	2.2 10 ⁻⁶
Al	3s ² 3p	3	2700	26.9815	1.8 10 ²²	2.7 10 ⁻⁶
Cu	3d ¹⁰ 4s	1	8960	63.546	8.5 10 ²²	1.7 10 ⁻⁶
Si	3s ² 3p ²	4	2330	28.086	20 10 ²²	6.2500

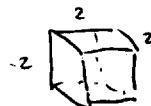
Kittel B.IMAS B.IMAS B.IMAS

- (2) Al and Si form sp hybridization. All 3s and 3p electrons contribute to valence. The valence can be determined from the column number in the periodic table of Mendeleev

$$(5) : n = v \times \frac{d}{w} \times N_A \quad N_A = 6.022 \cdot 10^{23}$$



2)



2 cm x 2 cm x 2 cm

$$\rho = 1 \Omega \text{ cm}, \quad \gamma_n = 1500 \text{ cm}^2/\text{Vs}, \quad V = 1 \text{ V}$$

a) $R = \rho \cdot \frac{L}{A} = 0.01 \Omega \text{ cm} \cdot \frac{0.02 \text{ m}}{(0.02)^2 \text{ m}^2} = 0.5 \Omega$

b) - $I = V/R = 1 \text{ V} / 0.5 \Omega = 2 \text{ A}$

- $j = I/A = 5000 \text{ A/m}^2$

c) $\bar{v} \equiv \gamma \cdot E = \gamma \frac{V}{L} = 1500 \text{ cm}^2/\text{Vs} \cdot \frac{1 \text{ V}}{2 \text{ cm}}$
 \uparrow
 sebenarnya, p. 6
 $= 750 \text{ cm/s}$
 $= 7.5 \text{ m/s}$

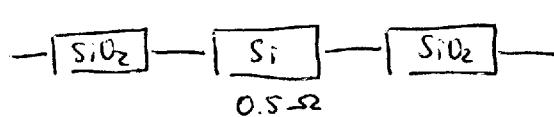
d) $j = q \cdot \gamma_n \cdot n \cdot E$

$$n = \frac{j}{q \gamma_n E} = \frac{j \cdot L}{q \gamma_n \cdot V} = \frac{5000 \text{ A/m}^2 \cdot 0.02 \text{ m}}{1.6 \cdot 10^{-19} \text{ C} \cdot 1500 \text{ cm}^2/\text{Vs} \cdot 1 \text{ V}}$$

$$= 4.2 \cdot 10^{21} \text{ /m}^3$$

$$= 4.2 \cdot 10^{15} \text{ /cm}^3$$

with 1 μm of SiO_2 :



$$\text{SiO}_2: \rho = 10^{15} \Omega \text{ cm}$$

$$= 10^{13} \Omega \text{ m}$$

$$R_{\text{SiO}_2} = \rho \cdot \frac{L}{A} = 10^{13} (\Omega \text{ m}) \cdot \frac{10^{-6} \text{ (m)}}{4 \cdot 10^{-4} \text{ (m}^2)} = 2.5 \cdot 10^{10} \Omega$$

$$R_{\text{tot}} = 2.5 \cdot 10^{10} + 0.5 + 2.5 \cdot 10^{10} = 5 \cdot 10^{10} \Omega$$

$$I = \frac{V}{R} = 2 \times 10^{-11} \text{ A} \quad (\text{compare answer 2b})$$

FCE

Problem sheet 4

Band model.

1) Calculate the density of states $g(E)$ of free electrons. Show a plot of $g(E)$ vs. E . Make sure that the units are $1/(m^3 J)^*$, remember Einstein's $E = mc^2$, thus $J = \text{kg m}^2/\text{s}^2$. Hint: The number of k values at an interval dk around a certain value k is $4\pi k^2 dk$.

2) Consider copper (Cu), which is a material in which each atom contributes a single electron to the bands ("valence" is 1, see problem sheet 3). Draw a schematic picture of the bands and their filling. On basis of this band model, would you classify copper as a metal, semimetal, semiconductor or insulator? Is this in line with your experience? What about silicon?

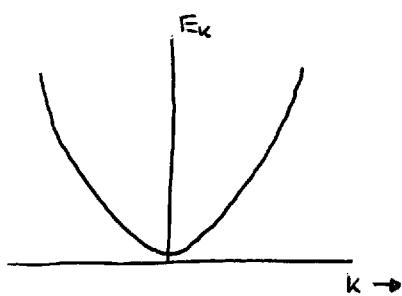
Which model is better: The Drude model, or the band model?

3) Explain how it is possible that Ga and As are metals, while, when mixed in a 50%-50% ratio, the result is a semiconductor.

*: more common is the unit $1/(cm^3 eV)$ where eV is the energy an electron gains in a voltage drop of 1 V. This energy is therefore equal to q Coulomb times 1 Volt, $1.6 \times 10^{-19} \text{ CV} = 1.6 \times 10^{-19} \text{ J}$.

Solutions to problem sheet 4

1)



$$E = \frac{\hbar^2 k^2}{2 m_e}$$

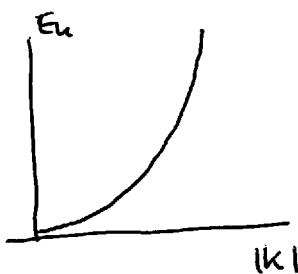
$$k = \sqrt{\frac{2 m_e E}{\hbar^2}}$$

Density of states is the number of available electronic states (per unit volume) per unit energy around an energy E . In other words it is the number of states ("k values") in a unit volume (in an energy interval dE) around E .

Since free electrons do not have k quantized, we cannot count them, but have to use the above idea

$$g(E) = \frac{\text{"volume of k values"}}{\text{"volume of energy"}} \quad \text{instead of} \quad g(E) = \frac{\text{"number of k values"}}{\text{"volume of energy"}}$$

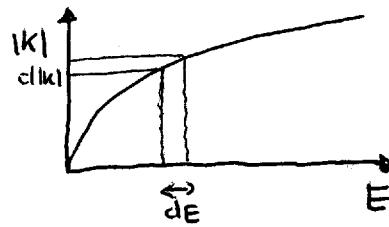
The "volume of energy" is just the interval of energy dE . The "volume of k values" depends on the dimension we are analyzing. For linear space (1 dimension), this "volume" is just the interval dk :



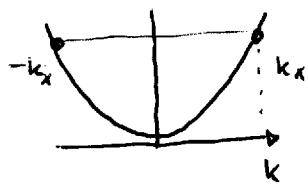
$E_k = \frac{\hbar^2 |k|^2}{2 m_e}$ is the energy as a function of k-value.

$|k| = \sqrt{\frac{2meE}{\hbar^2}}$ is the wave number as a function of energy

$$\frac{d|k|}{dE} = \sqrt{\frac{m_e}{2\hbar^2 E}}$$

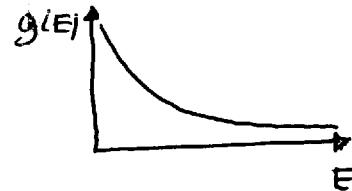


is the "volume of k values" per "volume of energy". We have to multiply this by 2 to get the density of states $g(E)$, because for every k positive there exists a k negative:



In one-dimensional space, $g(E)$ becomes therefore:

$$g(E) = 2 \sqrt{\frac{m_e}{\hbar^2 E}}$$



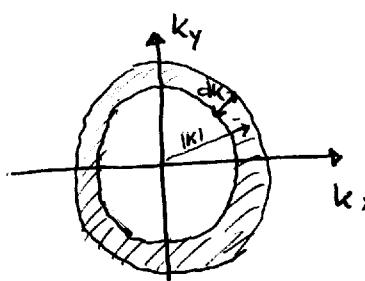
Checking the units: $[m_e] = \text{kg} = \text{J s}^2/\text{m}^2$ (Einstein $E=mc^2$)

$$[\hbar] = \text{Js}$$

$$[E] = \text{J}$$

$$[g(E)] = \sqrt{\text{Js}^2/\text{m}^2 \text{J}^2 \text{s}^2 \text{J}} = 1/\text{mJ} \quad \text{correct (1 dim.)}$$

If the electron is moving in 2 dimensions, the "volume of k values" at $|k|$ is no longer $d|k|$, but instead the area of the shaded zone in the figure



below. area = $2\pi|k|dk$.

Instead of using the $d|k|$ in the equations above we now have to use $2\pi|k|dk$:

$$g(E) = \frac{dk}{dE} = \sqrt{\frac{2m_e}{\hbar^2 E}}$$

becomes

$$g(E) = \frac{2\pi |k| dk/dE}{dE} = 2\pi |k| \cdot \frac{dk}{dE} = 2\pi |k| \sqrt{\frac{2m_e}{\hbar^2 E}}$$

$$= 2\pi \cdot \sqrt{\frac{2m_e E}{\hbar^2}} \cdot \sqrt{\frac{2m_e}{\hbar^2 E}} = 4\pi \frac{m_e}{\hbar^2}$$

checking units :

$$[g(E)] = \frac{kg}{J^2 s^2} = \frac{Js^2/m^2}{J^2 s^2} = \frac{1}{J m^2} \text{ correct (2 dim.)}$$

Now 3 dimensions :

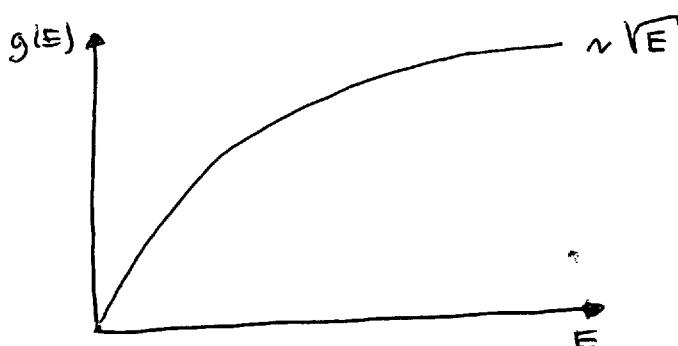
Instead of the volume $|dk|$ we now have $4\pi k^2 (dk)$

$$g(E) = \frac{4\pi k^2 dk/dE}{dE} = 4\pi |k|^2 \cdot \frac{dk}{dE} = 4\pi |k|^2 \cdot \sqrt{\frac{2m_e}{\hbar^2 E}}$$

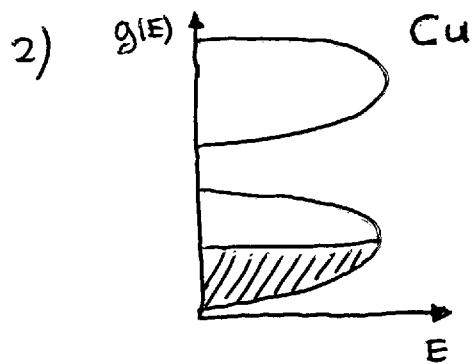
$$= 4\pi \left(\sqrt{\frac{2m_e E}{\hbar^2}} \right)^2 \cdot \sqrt{\frac{2m_e}{\hbar^2 E}} = 4\pi (m_e^{3/2}) \cdot \sqrt{E} / \hbar^3$$

Checking units :

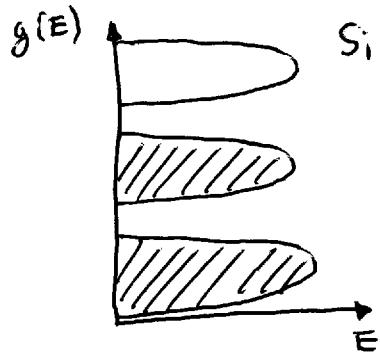
$$[g(E)] = \frac{(kg)^{3/2} \cdot J^{1/2}}{J^3 s^3} = \frac{J^{3/2} \cdot s^3/m^3 J^{1/2}}{J^3 s^3} = \frac{1}{J m^3} \text{ correct!}$$



If we include spin, all $g(E)$'s should be multiplied by 2



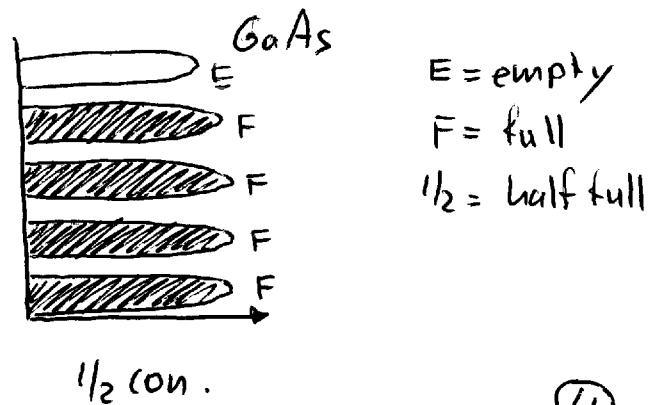
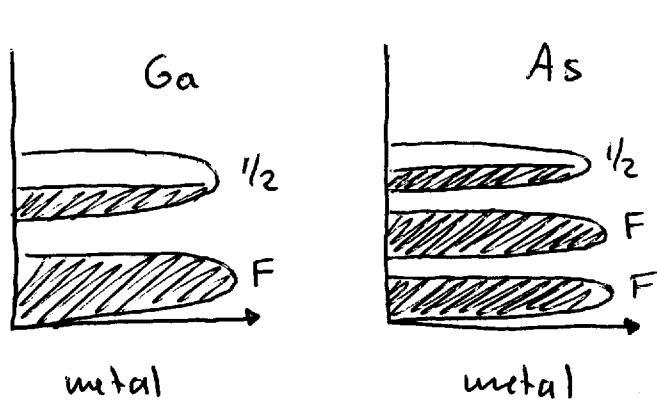
Number of states in each band is $2N$, with N the number of units (atoms in this case) in the sample. Each unit (atom) contributes 1 (one) electron to the conduction. Therefore, the number of electrons is N and the band is half full (half empty). Therefore, Cu is a metal



In Si each unit (atom) contributes 4 electrons. These fill up two bands and therefore, the material is either a semiconductor, a semimetal or an insulator.

This band model predicts a low conductivity for Si and is thus better than the Drude model

3) GaAs : In each unit (combination of Ga and As) there are 8 electrons (3 from Ga and 5 from As). This fills up 4 bands \rightarrow semiconductor

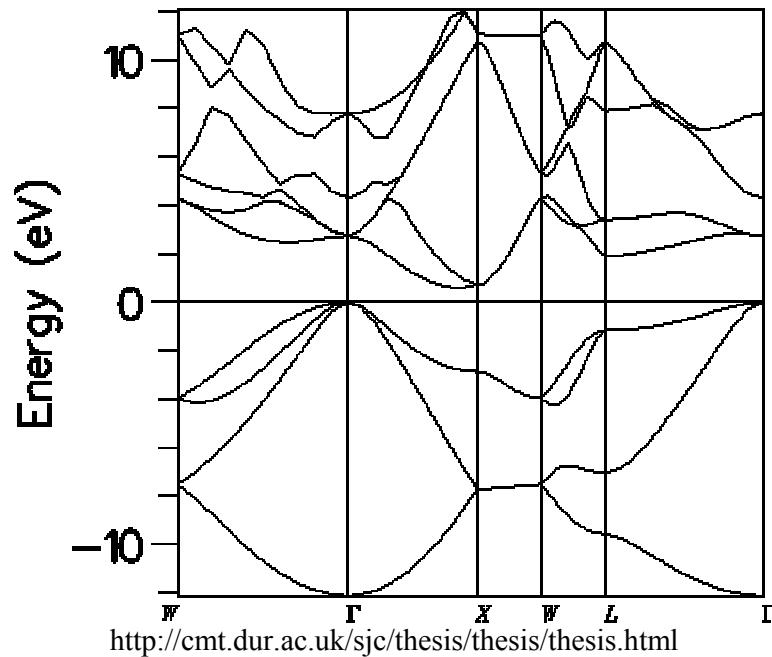


FCE

Problem sheet 5

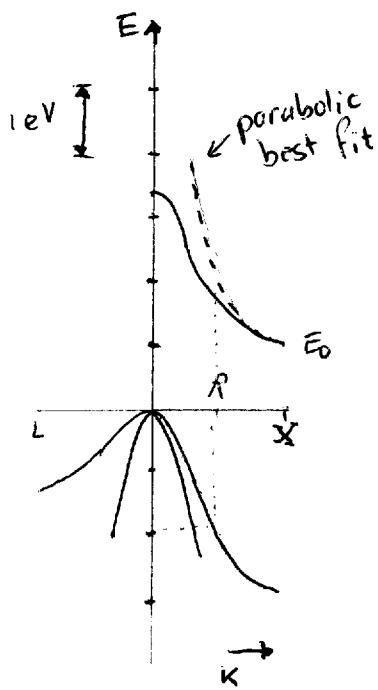
Band structure II.

- 1) Give an estimate for the effective mass of electrons and holes in silicon, based on the figure on p. 25 of the lecture notes (or figure below).
- 2) What is the maximum wavelength that can be used to induce a direct transition of an electron from the (top of) the valence band to the conduction band in silicon?
- 3) What is the total density of states in the valence band of silicon and what is the effective density of states? What % is that?



Solutions to problem sheet 5

1)



$$X = \frac{2\pi}{a} = 1.16 \cdot 10^{10} / m$$

$$a_{Si} = 5.43095 \text{ \AA} \quad (\text{appendix H})$$

The dashed line --- shows a parabolic best fit. At a distance halfway (point R), the parabola has risen appr. 1.3 eV. This is enough to calculate the effective mass:

$$E_R = \frac{\hbar^2 (k_R - k_0)^2}{2 m_e^*} + E_0 \quad k_0 = \frac{2\pi}{a} = 1.16 \cdot 10^{10} / m$$

$$k = k_R = \frac{1}{2} \left(\frac{2\pi}{a} \right) = \frac{\pi}{a} = 0.58 \cdot 10^{10} / m$$

$$E_R = E_0 + 1.3 \text{ eV}$$

$$1.3 \cdot 1.6 \cdot 10^{-19} \text{ (5)} = \frac{\hbar^2 (0.58 \cdot 10^{10})^2}{2 m_e^*} \Rightarrow m_e^* = 9.0 \cdot 10^{-30} \text{ kg}$$

$$= 0.99 m_0$$

—

(appendix H: 0.98)

holes: $E_R \approx -2 \text{ eV}$

$$E_R = - \frac{\hbar^2 (k_h - k_0)^2}{2 m_h^*} + E_0 \quad E_0 = 0$$

$$k_0 = 0$$

$$k_R = \pi/a = 0.58 \cdot 10^{10}$$

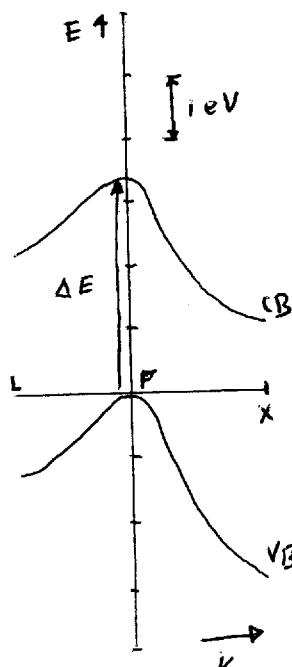
$$-2 \cdot 1.6 \cdot 10^{-19} \text{ (5)} = \frac{\hbar^2 (0.58 \cdot 10^{10})^2}{2 m_h^*} \Rightarrow m_h^* = 5.85 \cdot 10^{-31} \text{ kg}$$

$$= 0.64 m_0$$

(appendix H: 0.16) 1

2)

Direct transition



Energy in transition

$$\Delta E \approx 3.3 \text{ eV}$$

$$E_{\text{photon}} = h\nu \Rightarrow \nu = \frac{3.3 \times 1.6 \times 10^{-19} \text{ J}}{6.626 \times 10^{-34} \text{ Js}}$$

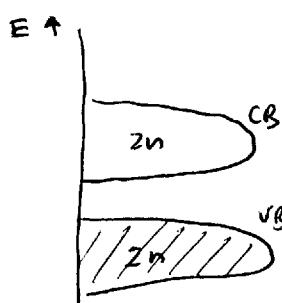
$$= 7.97 \times 10^{14} \text{ Hz}$$

$$\lambda = \frac{c}{\nu} = \frac{3 \cdot 10^8 \text{ m/s}}{7.97 \times 10^{14} \text{ /s}} = 376 \text{ nm}$$

 \Rightarrow ultra violet!

- 3) Total number (density) of states in valence band = $2n$, with n the density of atoms
 n can be found in appendix H:

$$n = 5.0 \times 10^{22} / \text{cm}^3$$

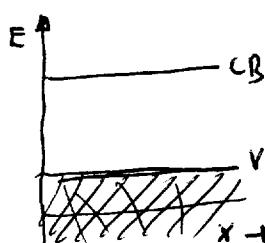


$$\text{d.o.s of VB: } 10^{23} / \text{cm}^3$$

$$\text{CB: } 10^{23} / \text{cm}^3$$

effective density of states : See appendix H

$$N_V = 1.04 \times 10^{19} / \text{cm}^3$$



$$\frac{N_V}{2n} = 0.0001 \quad (0.01\%)$$

$$N_C = 2.8 \times 10^{19} / \text{cm}^3$$

$$\frac{N_C}{2n} = 0.0003 \quad (0.03\%)$$

Effectively only a very very very small edge of the bands contribute to conduction processes : 0.01% of appr. 4 eV = 0.4 eV

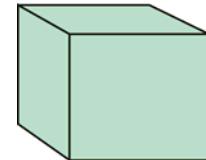
FCE

Problem sheet 6

Crystals

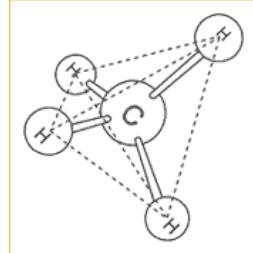
1) Find the 5 different Bravais lattices of two-dimensional crystals.

2) What are the (24) symmetry operations of a cube? Note: do not count operations that can be constructed from others. For example: a rotation through an axis over 90° is the same as one of -90° because the latter is the same as three times executing the first.

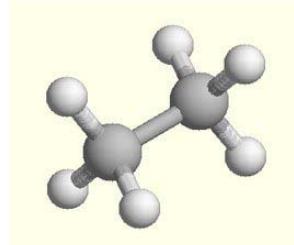


3) List the symmetry operations of a molecule of

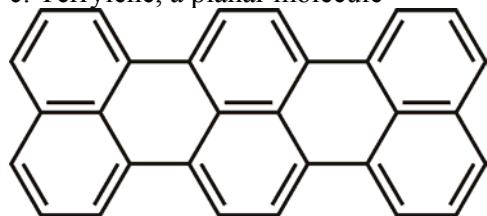
a: Methane



b: Ethane



c: Terrylene, a planar molecule



Solutions to problem sheet 6

Two-Dimensional Lattice Types

There is an unlimited number of possible lattices because there is no natural restriction on the lengths a, b of the lattice translation vectors or on the angle φ between them. The lattice in Fig. 7a was drawn for arbitrary \mathbf{a} and \mathbf{b} . A general lattice such as this is known as an **oblique lattice** and is invariant only under rotation of π and 2π about any lattice point.

But special lattices of the oblique type can be invariant under rotation of $2\pi/3$, $2\pi/4$, or $2\pi/6$, or under mirror reflection. We must impose restrictive conditions on \mathbf{a} and \mathbf{b} if we want to construct a lattice which will be invariant under one or more of these new operations. Such restrictions are found below: there are four distinct types of restriction, and each leads to what we may call a **special lattice type**.

Thus there are five distinct lattice types in two dimensions, the oblique lattice and the four special lattices. **Bravais lattice** is the common phrase¹¹ for a distinct lattice type; we say that there are five Bravais lattices in two dimensions.

The point operation 4 requires a square lattice (Fig. 13a). The point operations 3 and 6 require a hexagonal lattice (Fig. 13b). This lattice is invariant under a rotation $2\pi/6$ about an axis through a lattice point and normal to the plane.

There are important consequences if the mirror reflection m is present. We write the primitive translation vectors \mathbf{a}, \mathbf{b} in terms of the unit vectors $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ along the Cartesian x, y axes:

$$\mathbf{a} = a_x \hat{\mathbf{x}} + a_y \hat{\mathbf{y}} ; \quad \mathbf{b} = b_x \hat{\mathbf{x}} + b_y \hat{\mathbf{y}} . \quad (4)$$

If the primitive vectors are mirrored in the x axis, then \mathbf{a}, \mathbf{b} are transformed by the reflection operation into new vectors \mathbf{a}', \mathbf{b}' given by

$$\mathbf{a}' = a_x \hat{\mathbf{x}} - a_y \hat{\mathbf{y}} ; \quad \mathbf{b}' = b_x \hat{\mathbf{x}} - b_y \hat{\mathbf{y}} . \quad (5)$$

If the lattice is invariant under the reflection, then \mathbf{a}', \mathbf{b}' must be lattice vectors; that is, they must be of the form $n_1 \mathbf{a} + n_2 \mathbf{b}$, where n_1 and n_2 are integers. If we take

$$\mathbf{a} = a \hat{\mathbf{x}} ; \quad \mathbf{b} = b \hat{\mathbf{y}} ; \quad (6)$$

then $\mathbf{a}' = \mathbf{a}$ and $\mathbf{b}' = -\mathbf{b}$, so that the lattice is carried into itself. The lattice defined by (6) is rectangular (Fig. 13c).

A second distinct possibility for \mathbf{a} and \mathbf{b} gives another type of lattice invariant under reflection. Note that \mathbf{b}' will be a lattice vector if

$$\mathbf{b}' = \mathbf{a} - \mathbf{b} ; \quad (7)$$

then using (5) we have

$$b'_x = a_x - b_x = b_x ; \quad b'_y = a_y - b_y = -b_y . \quad (8)$$

These equations have a solution if $a_y = 0$; $b_x = \frac{1}{2}a_x$; thus a possible choice of primitive translation vectors for a lattice with mirror symmetry is

$$\mathbf{a} = a \hat{\mathbf{x}} ; \quad \mathbf{b} = \frac{1}{2}a \hat{\mathbf{x}} + b \hat{\mathbf{y}} . \quad (9)$$

This choice gives a centered rectangular lattice (Fig. 13d).

We have now exhausted the two-dimensional Bravais lattices which are consistent with the point-group operations applied to lattice points. The five possibilities in two dimensions are summarized in Table 1. The point symmetry given is that of the *lattice*; an actual crystal structure may have lower symmetry than its lattice. Thus it is possible for a crystal with a square lattice to have the operation 4 without having all of the operations 4mm.

extract from
Kittel, "Solid
State Physics"

¹¹ We have not succeeded in finding or constructing a definition which starts out "A Bravais lattice is . . ."; the sources we have looked at say "That was a Bravais lattice." The phrase "fundamental type of lattice" is more suggestive.

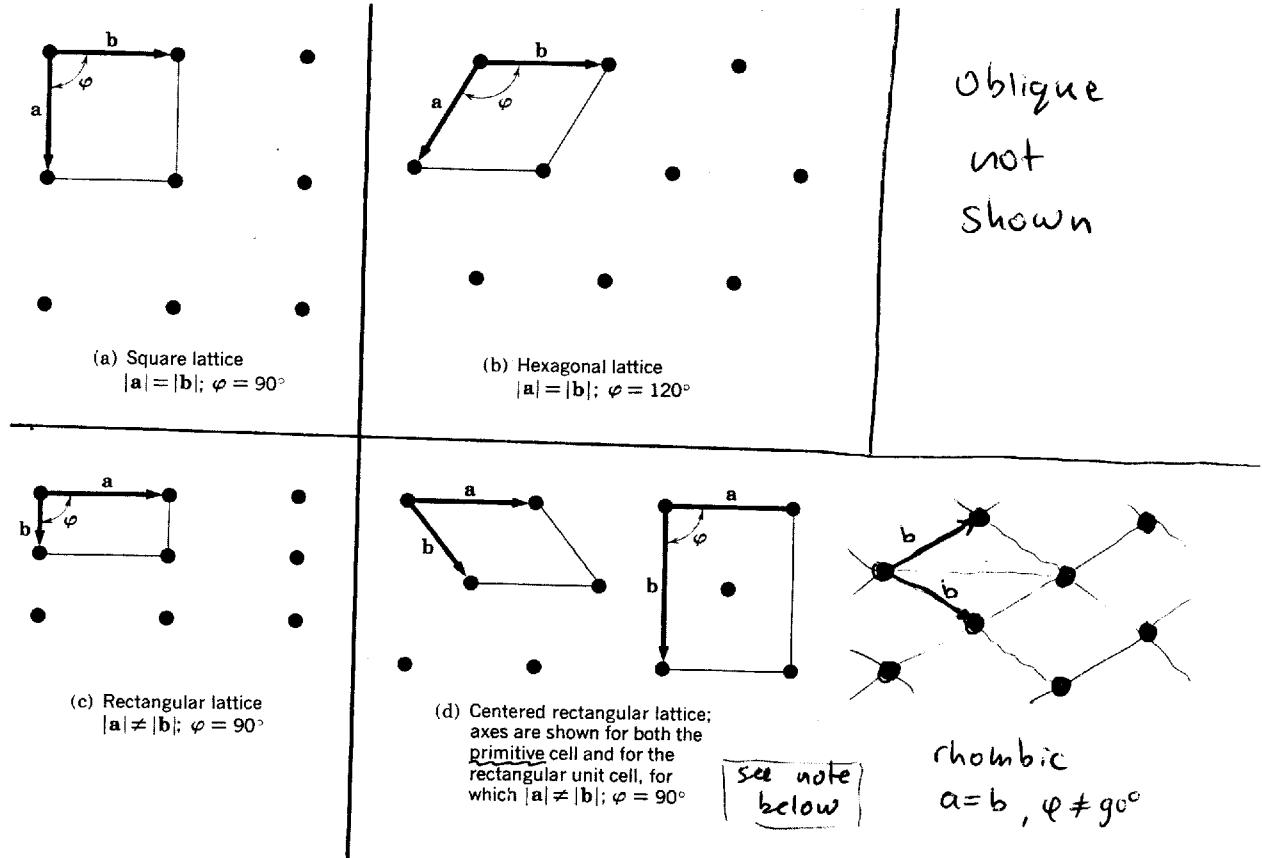


Figure 13

Table 1 The five two-dimensional lattice types
(The notation mm means that two mirror lines are present)

Lattice	Conventional cell	Axes of conventional cell	Point-group symmetry of lattice about lattice points
Fig. 13a ← Oblique	Parallelogram	$a \neq b, \varphi \neq 90^\circ$	2
13b ← Square	Square	$a = b, \varphi = 90^\circ$	4mm
13c ← Hexagonal	60° rhombus	$a = b, \varphi = 120^\circ$	6mm
13d ← Primitive rectangular	Rectangle	$a \neq b, \varphi = 90^\circ$	2mm
13d ← Centered rectangular	Rectangle	$a \neq b, \varphi = 90^\circ a=b, \varphi \neq 90^\circ$	2mm

Note: "Primitive" means "the smallest possible"

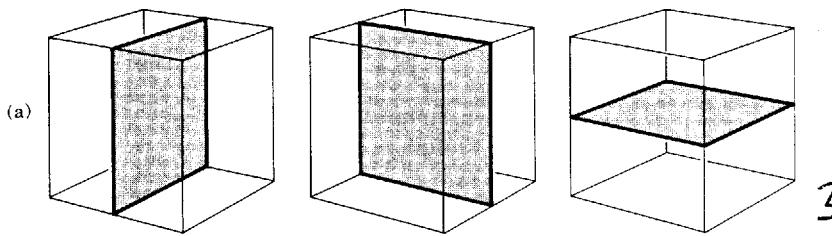
name	symbol	number	
unity	E	1x	
inversion	I	1x	
mirror (a)	m	3x	
mirror (b)	m	6x	
90° rotation	R_4	3x	
120° rotation	R_3	4x	
180° rotation	R_2	6x	

total 24

$R_2 \times m = I$
 $\begin{pmatrix} x \rightarrow -x \\ y \rightarrow -y \\ z \rightarrow z \end{pmatrix} \times \begin{pmatrix} x \rightarrow x \\ y \rightarrow y \\ z \rightarrow -z \end{pmatrix} = \begin{pmatrix} x \rightarrow -x \\ y \rightarrow -y \\ z \rightarrow -z \end{pmatrix}$

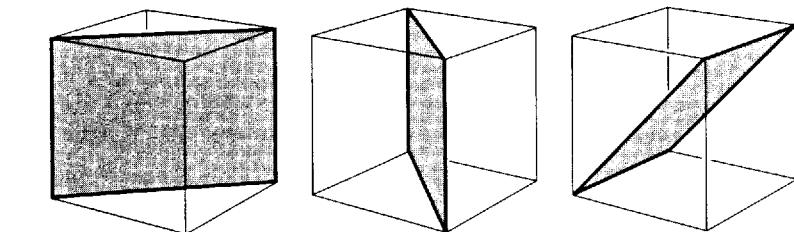
if we include also
improper rotations and inverses

FP 6 (2)

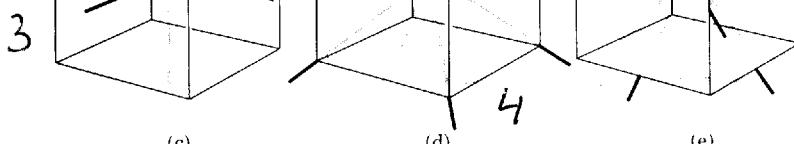
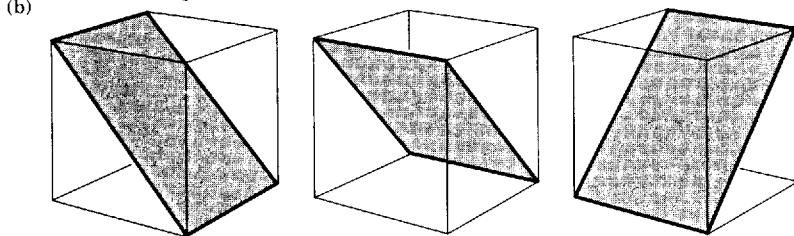


← Kittel, p. 15

3



6



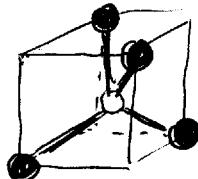
4



6

Figure 12 (a) The three planes of symmetry parallel to the faces of a cube. (b) The six diagonal planes of symmetry in a cube. (c) The three tetrad axes of a cube. (d) The four triad axes of a cube. (e) The six diad axes of a cube. The inversion center is not shown.

3) a)



$4 \times R_3$ (fig. 12 d)

$3 \times R_2$ (fig. 12 c)

$6 \times m$ (fig. 12 b)

$1 \times E$

"improper axes" $\rightarrow 3 \times S_4$ (fig. 12 c, $R_4 * I$)

not:	R_2 of fig. 12 e
	m of fig. 12 a
	I

3 b)

Ethane

Symmetry elements	
$1 \times E$	
$1 \times S_6 (= C_6 \times m)^*$	$3 \times m$ (Containing C_3) for example paper plane
$1 \times C_3$	$3 \times C_2$
$1 \times I$	

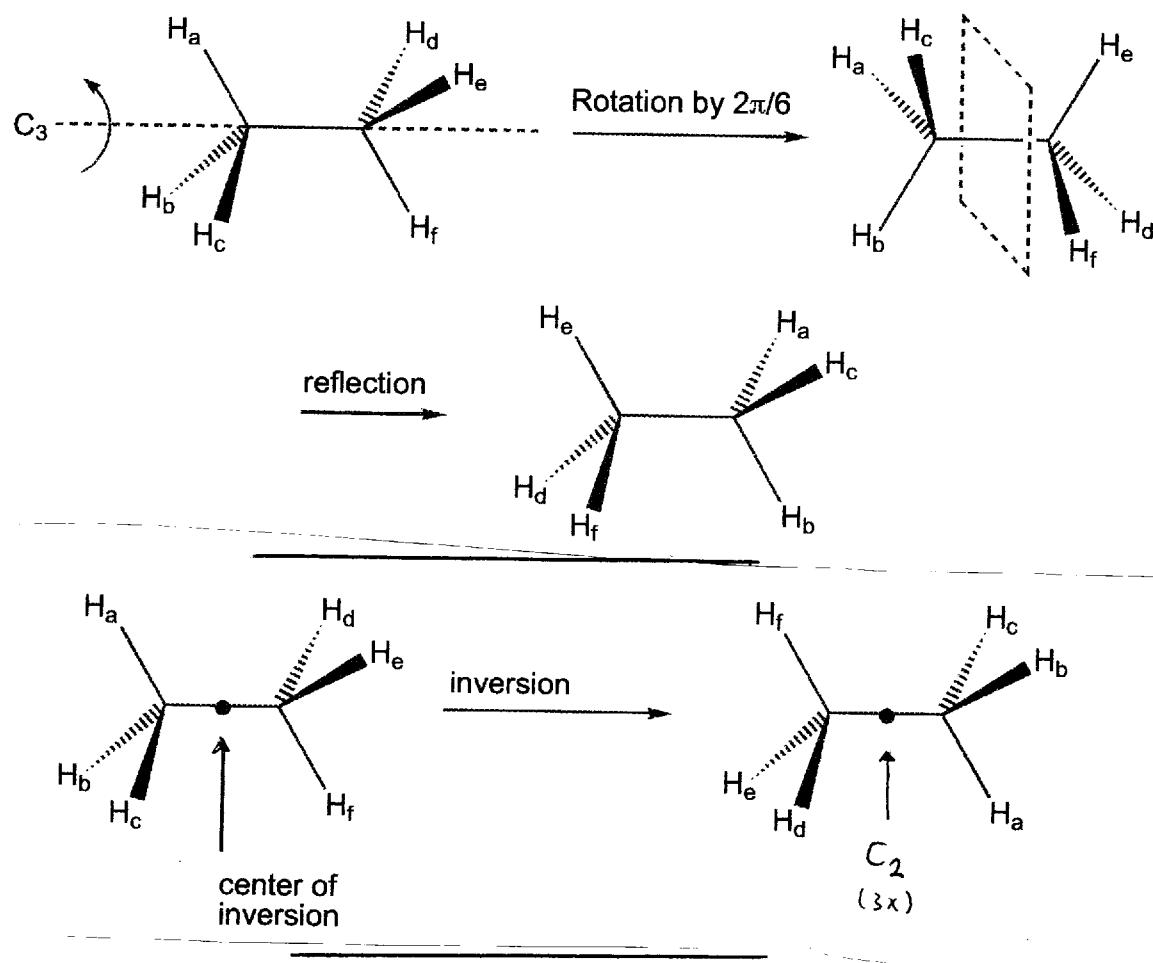
$$* S_6 = C_6 \times m$$

or

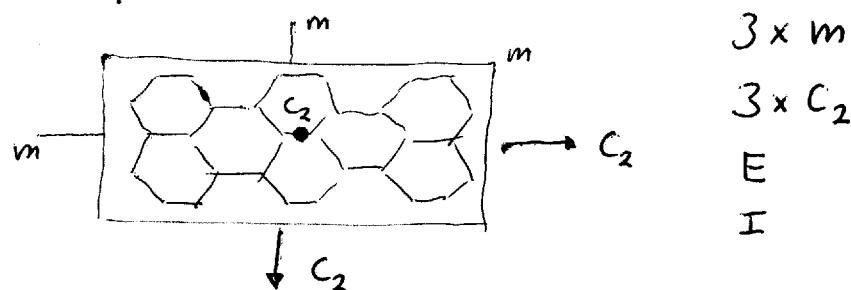
$$S_3 = C_3 \times I$$

<http://web.mit.edu/5.03/www/-notes/04.pdf>

Pictures from



3 c) Terrylene



FCE

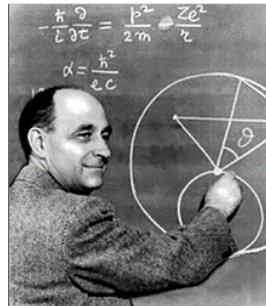
Problem sheet 7

Fermi level

- 1) What is the position of the Fermi level in GaAs at room temperature (300 K)? (Define the top of the valence band as 0).
- 2) What is the energy range where the Fermi-Dirac distribution drops from “levels 90% occupied” to “levels 10% occupied”? ($T = 300 \text{ K}$).



Paul Dirac



Enrico Fermi

Solution to

Problem Sheet 7

$$1) E_F = \frac{kT}{2} \ln \left(\frac{N_V}{N_C} \right) + \left(\frac{E_V + E_C}{2} \right)$$

$$N_V = 7.0 \cdot 10^{18}$$

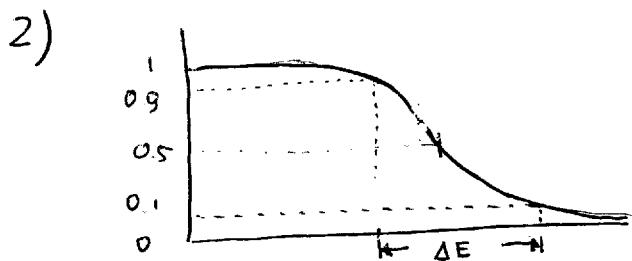
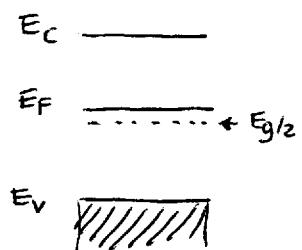
$$N_C = 4.7 \cdot 10^{17}$$

$$E_V = 0$$

$$E_C = E_V + E_g = 1.424 \text{ eV}$$

$$T = 300 \text{ K}, \quad kT = 25.8 \text{ meV}$$

$$E_F = 34.9 \text{ meV} + \frac{1.424 \text{ eV}}{2} = 0.747 \text{ eV}$$



$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$$f(E) = 0.9$$

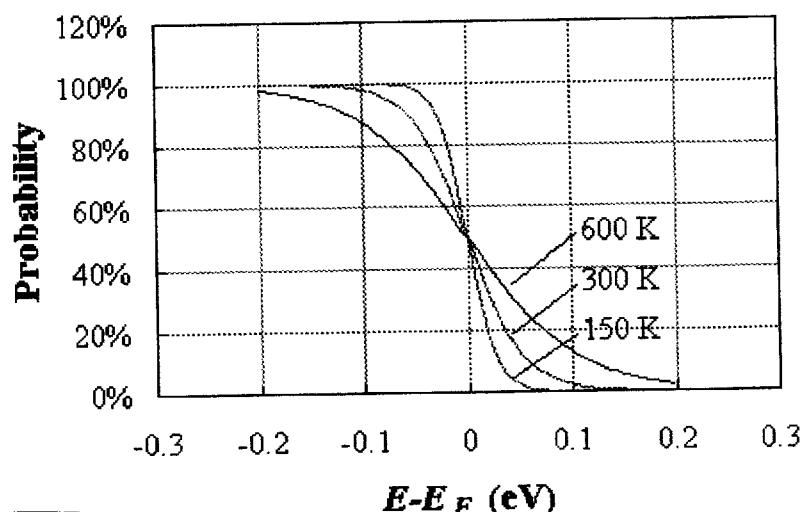
$$\frac{E - E_F}{kT} = \ln\left(\frac{1 - 0.9}{0.9}\right)$$

$$E = E_F + kT \times \ln\left(\frac{1 - 0.9}{0.9}\right) = \\ = E_F - 57 \text{ meV}$$

$$f(E) = 0.1$$

$$E = E_F + kT \times \ln\left(\frac{1 - 0.1}{0.1}\right) = E_F + 57 \text{ meV}$$

$$\Delta E = \\ 114 \text{ meV}$$



FCE

Problem sheet 8

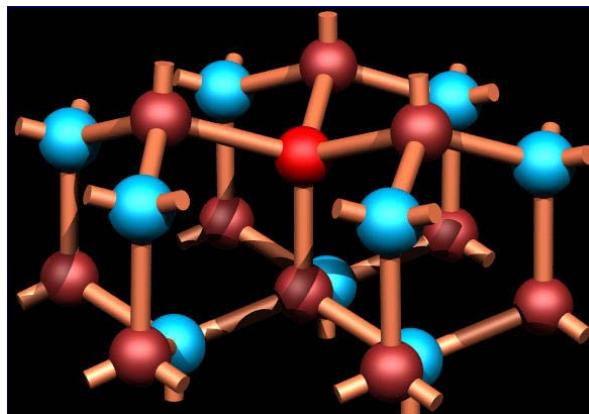
Fermi level II. Doping

- 1a) Determine the position of the Fermi level E_F in GaAs doped with an acceptor.

E_V	0 (by definition)
E_A	100 meV
N_A	$10^{15} / \text{cm}^3$
T	300 K

Determine the density of electrons in the conduction band.

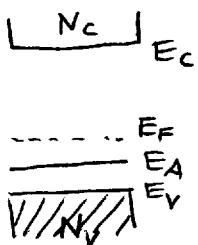
- 1b) Draw schematic pictures of E_F and conductivity σ (assuming constant mobility μ) as a function of temperature.



- 2) Calculate the depth of acceptor and donor levels in GaAs on basis of the effective-mass theory.

Solution to FP 8 , FCE

1a)



$$E_V \equiv 0$$

$$E_A = 100 \text{ meV}$$

$$N_A = 10^{15} / \text{cm}^3$$

$$T = 300 \text{ K}$$

appendix H:

$$N_c = 4.7 \cdot 10^{17}$$

$$N_v = 7.0 \cdot 10^{18}$$

$$E_g = 1.424 \text{ (300K)}$$

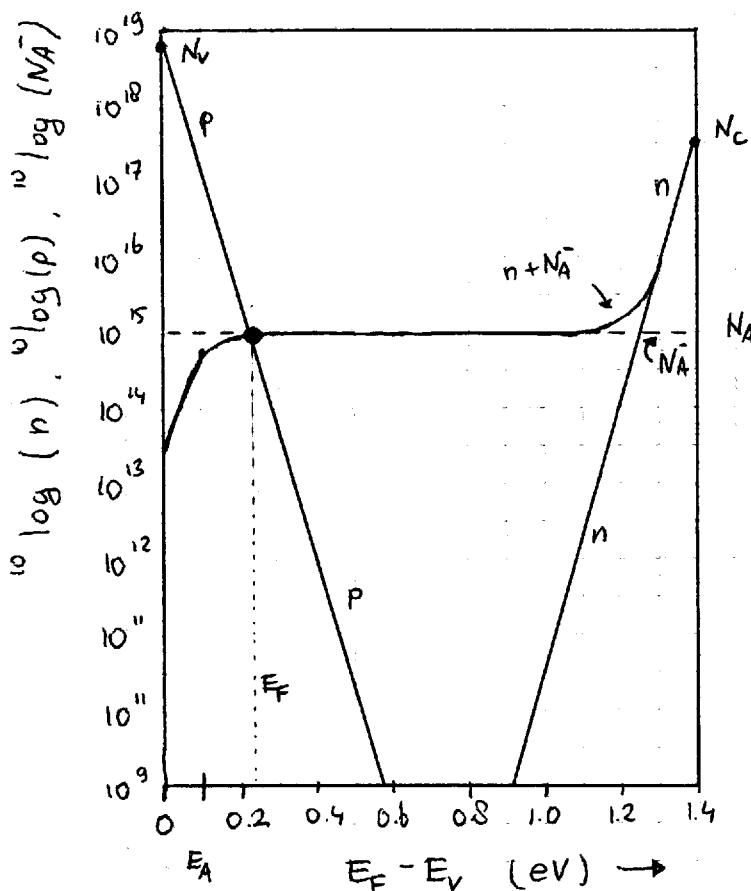
$$n + N_A^- = P \quad (0)$$

(charge neutrality : density of free electrons + density of ionized acceptors must be equal to density of hole in valence band)

$$n = f(E) \cdot N_c \approx N_c \cdot \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$P = (1 - f(E)) \cdot N_v \approx N_v \cdot \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$N_A^- = f(E) \cdot N_A = N_A \cdot \frac{1}{1 + \exp\left(\frac{E_A - E_F}{kT}\right)}$$



- $E_F \approx 0.23 \text{ eV}$

(using graphical method)

or:
• Approximation :

$$n = 0 \quad (I)$$

$$P = N_v \exp\left(\frac{E_v - E_F}{kT}\right) \quad (II)$$

$$N_A^- = N_A \quad (III)$$

$$I + II + III \text{ in } (0)$$

$$E_F = kT \ln \left(\frac{N_v}{N_A} \right)$$

$$= 0.229 \text{ eV}$$

- Density of free electrons:

First determine density of free holes:

$$p \sim N_A = 10^{15} \text{ cm}^{-3} \quad (\text{see Figure on prev. page})$$

Then, remember that independent of doping

$$n \cdot p = N_c \cdot N_v \cdot \exp\left(-\frac{E_g}{kT}\right)$$

$$n = \frac{N_c \cdot N_v}{p} \exp\left(-\frac{E_g}{kT}\right)$$

$$= \frac{4.7 \cdot 10^{17} \text{ cm}^{-3} \cdot 7.0 \cdot 10^{18} \text{ cm}^{-3}}{10^{15} \text{ cm}^{-3}} \cdot \exp\left(-\frac{1.424 \text{ eV}}{25.85 \text{ meV}}\right)$$

$$= 3.9 \cdot 10^{-3} \text{ cm}^{-3}$$

1b)

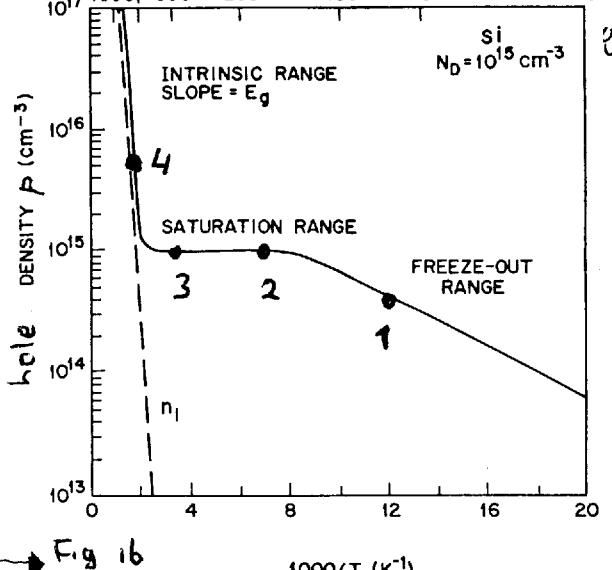
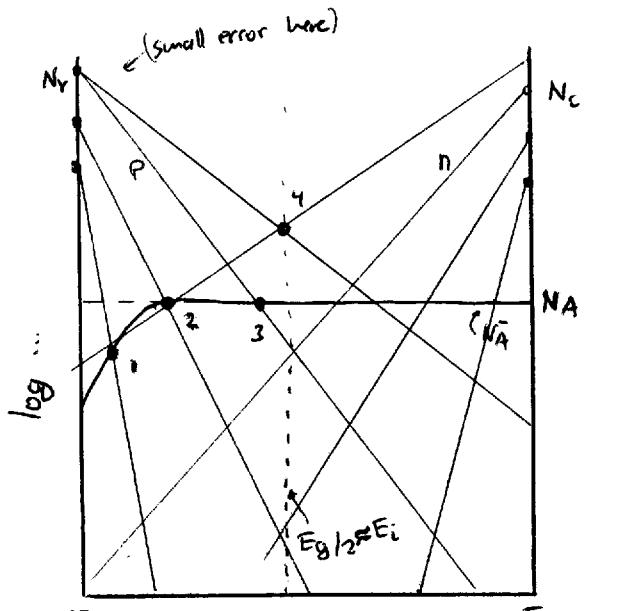
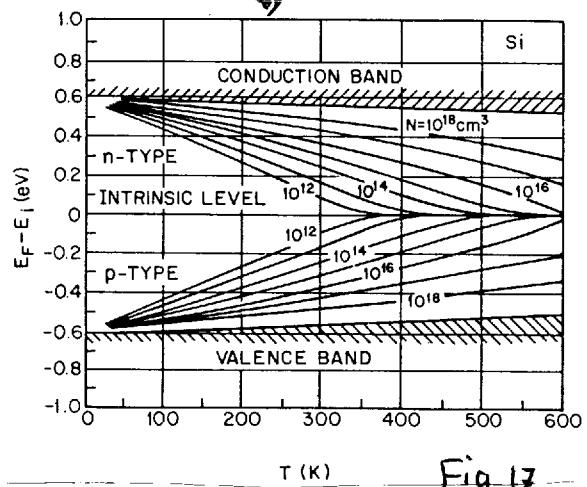
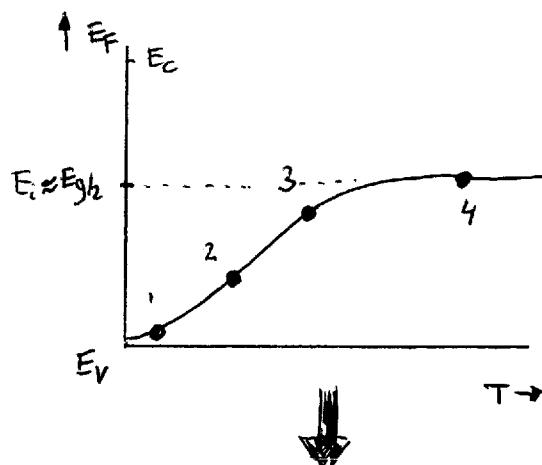


Fig. 16 hole density as a function of temperature for a Si sample with $N_D = 10^{15} \text{ cm}^{-3}$. Impurity concentration of 10^{15} cm^{-3} .

- (1) Slope changes
- (2) N_c and $N_v \sim T^{3/2}$



$$\rightarrow O \approx (n + p) \approx p$$

Figures of See.

Fig. 16

$1000/T \text{ (K}^{-1}\text{)}$

2) p. 46 of Sebenta:

$$E_H = 13.6 \text{ eV} \quad (\text{hydrogen atom})$$

$$E_d = E_H \frac{m_e^*}{m_e} \cdot \frac{1}{\epsilon_r^2} \quad \epsilon_r = 13.1$$

$$E_a = E_H \frac{m_h^+}{m_e} \cdot \frac{1}{\epsilon_r^2} \quad m_e^* = 0.067$$

$$m_h^+ = 0.082$$

$$m_h = 0.45$$

Appendix
H

$$E_d = 5.3 \text{ meV}$$

$$E_{a_1} = 6.0 \text{ meV}$$

$$E_{a_2} = 33.0 \text{ meV}$$

FCE

Problem sheet 9 (Diffusion an drift) currents, pn-junctions

- 1) Using Einstein's relation that couples the diffusion coefficient to the mobility:

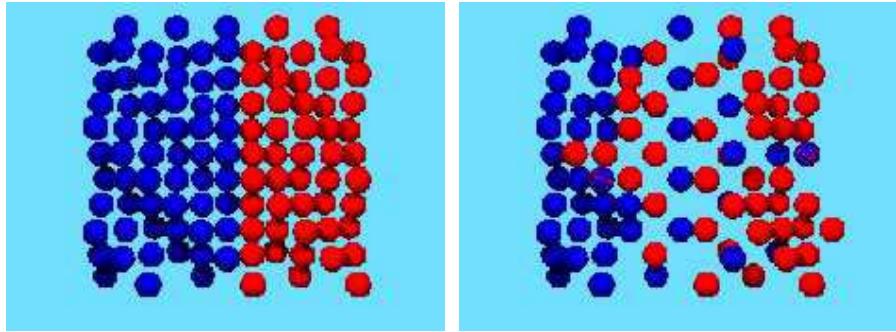
$$D_n = \mu_n kT/q$$

show that the current-density equation (CDE) of page 54 of the sebenta,

$$J_n = q \mu_n n E_x + q D_n dn/dx$$

reduces to the equation on page 56:

$$J_n = \mu_n n dE_F/dx$$



Diffusion of electrons (blue, dark, left side of image on the left) and holes (red, lighter, right side of image on the left) into each other. Left image: at the start, right image: after a while. (image from <http://www.chem.tue.nl/glasstech/diffusion/right.html>)

- 2) In a plot of a pn-junction (energy E vs. space x), indicate/explain

- The conduction and valence band.
- The Fermi level.
- The donor and acceptor levels.
- The diffusion and drift currents of electrons.

- 3) Calculate the built-in voltage of a pn-junction diode made of GaAs at $T = 300$ K. P-type side: $E_A = 100$ meV, $N_A = 10^{15}/\text{cm}^3$. N-type side: $E_D = 100$ meV, $N_D = 10^{15}/\text{cm}^3$.

Solution to FCE FPg

$$I) \quad J_n = q \gamma_n n E_x + q D_n \frac{dn}{dx} \quad (I)$$

$$D_n = \frac{kT}{q} \gamma_n \quad (\text{Einstein}) \quad (II)$$

I in II :

$$J_n = q \gamma_n n E_x + kT \gamma_n \frac{dn}{dx} \quad (III) \quad \begin{array}{c} E_c(x) \\ \Delta E \\ E_F(x) \end{array}$$

$$n = N_c \cdot \exp\left(-\frac{|E_c - E_F|}{kT}\right)$$

$$= N_c \cdot \exp\left(\frac{E_F(x) - E_c(x)}{kT}\right) = N_c \exp\left(\frac{E_F(x) - E_c(-\infty) + qV(x)}{kT}\right)$$

$$\frac{dn}{dx} = N_c \exp\left(\frac{E_F(x) - E_c(-\infty) + qV(x)}{kT}\right) \cdot \frac{1}{kT} \left(\frac{dE_F(x)}{dx} - q \frac{dV(x)}{dx} \right)$$

$$= n \cdot \frac{1}{kT} \left(\frac{dE_F(x)}{dx} - q \frac{dV(x)}{dx} \right)$$

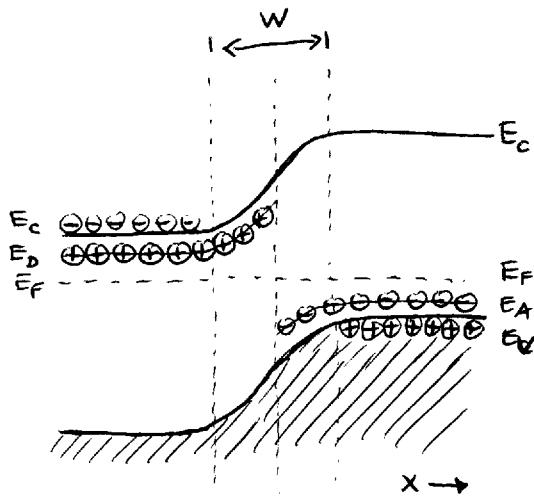
$$= n \cdot \frac{1}{kT} \left(\frac{dE_F(x)}{dx} - q \cdot E(x) \right) \quad \text{in (III):}$$

$$J_n = q \gamma_n n E_x + kT \gamma_n \left[n \frac{1}{kT} \left(\frac{dE_F(x)}{dx} - q E(x) \right) \right]$$

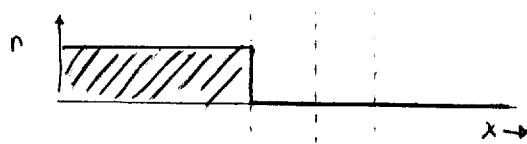
$$= \gamma_n n \left(q E_x + \frac{dE_F(x)}{dx} - q E_x \right)$$

$$= \gamma_n n \frac{dE_F(x)}{dx} \quad \underline{\text{q.e.d.}}$$

2)



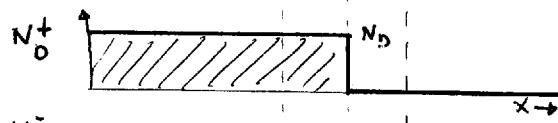
band diagram



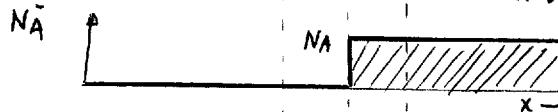
'free' electrons



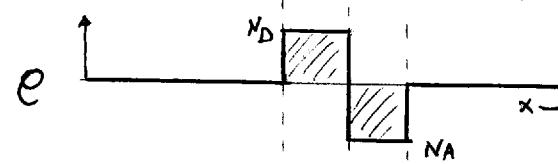
'free' holes



ionized donors



ionized acceptors



charge density

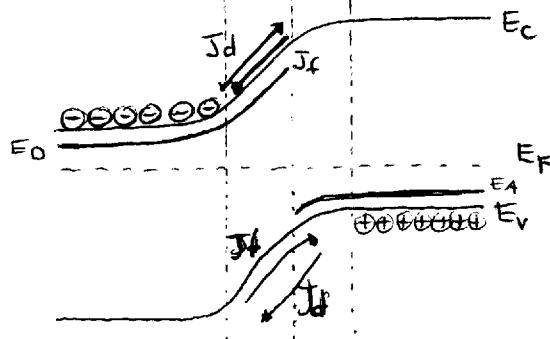
$$e = q(p - n + N_D^+ - N_A^-)$$

electric field

$$E = \int \frac{1}{\epsilon} \rho(x) dx$$

voltage

$$V = \int E(x) dx = \iint \frac{1}{\epsilon} \rho(x) dx^2$$

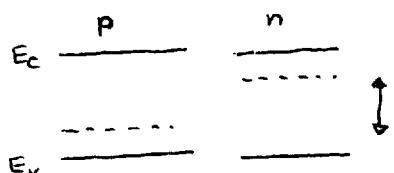
J_d: drift diffusion currentJ_f: drift current

3) see FP8 on how to determine the position of the Fermi level

$$p\text{-type: } E_F - E_A = kT \ln \left(\frac{N_V}{N_A} \right) = 0.22 \text{ eV}$$

(note E_A is not important under normal circumstances)

$$n\text{-type: } E_C - E_F = kT \ln \left(\frac{N_C}{N_D} \right) = 0.15 \text{ eV}$$



$$\begin{aligned} \Delta E_F &= E_g - 0.22 \text{ eV} - 0.15 \text{ eV} \\ &= 1.424 \text{ eV} - 0.22 \text{ eV} - 0.15 \text{ eV} \\ &= 1.036 \text{ eV} \end{aligned}$$

$$V_{bi} = \frac{1}{q} \Delta E_F = 1.036 \text{ V}$$

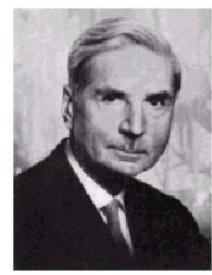
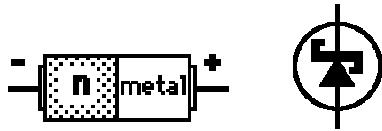
FCE

Problem sheet 10

Schottky barriers

- 1) Calculate the built-in voltage of Schottky barriers of gold and aluminum with p-type and n-type silicon ($T = 300 \text{ K}$, $N_D = 10^{15}/\text{cm}^3$, $N_A = 10^{15}/\text{cm}^3$).
- 2) What are the depletion width and the depletion capacitance of each of the four diodes above? (Active area: $0.1\text{mm} \times 0.1 \text{ mm}$).
- 3) The following data contain a measurement of the capacitance of a silicon Schottky diode as a function of bias (active area $1\text{mm} \times 1\text{mm}$). Determine the built-in voltage and the impurity concentration.

bias (V)	capacitance (F)
-5.0	1.30370786559725E-0010
-4.5	1.36066492495419E-0010
-4.0	1.42580494037567E-0010
-3.5	1.50129586146432E-0010
-3.0	1.59020558670971E-0010
-2.5	1.69704732246026E-0010
-2.0	1.82879218142384E-0010
-1.5	1.99691017216427E-0010
-1.0	2.22195651802606E-0010
-0.5	2.54557098368933E-0010
0.0	3.07007412003949E-0010
0.5	4.15690000938071E-0010
1.0	1.01822839347573E-0009

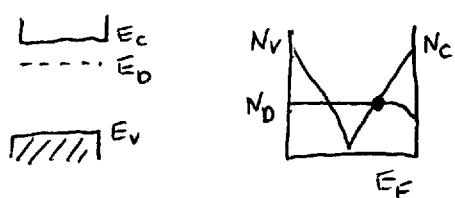


*Schottky: a) Cross section of diode b) Electronic symbol
c) Some examples d) Walter Schottky (1886-1976).*

Solution to FCE FP 10

1)

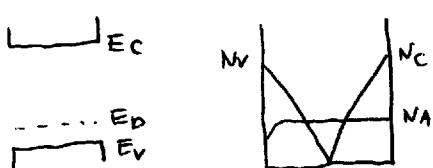
n-type silicon



$$N_c \exp\left(\frac{E_F - E_c}{kT}\right) = N_D$$

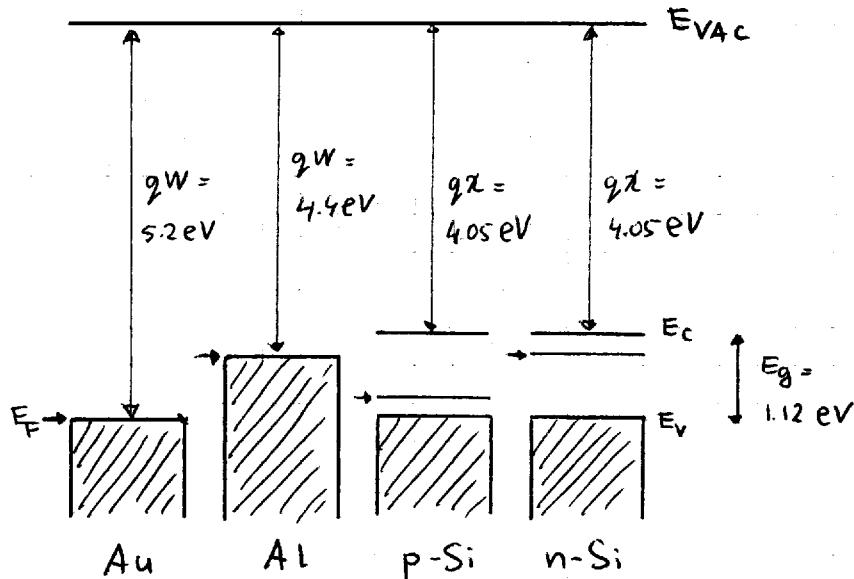
$$\begin{aligned} E_F &= -kT \ln \frac{N_c}{N_D} + E_c \\ &= E_c - 0.265 \text{ eV} \end{aligned}$$

p-type silicon



$$N_v \exp\left(\frac{E_v - E_F}{kT}\right) = N_A$$

$$\begin{aligned} E_F &= kT \ln\left(\frac{N_v}{N_A}\right) + E_v \\ &= E_v + 0.239 \text{ eV} \end{aligned}$$



$$\text{Au : } E_{VAC} - 5.2 \text{ eV}$$

$$\text{Al : } E_{VAC} - 4.4 \text{ eV}$$

$$\begin{aligned} \text{p-Si : } E_{VAC} - qX - E_g + 0.239 \text{ eV} \\ = E_{VAC} - 4.931 \text{ eV} \end{aligned}$$

$$\begin{aligned} \text{n-Si : } E_{VAC} - qX - 0.265 \text{ eV} \\ = E_{VAC} - 4.315 \text{ eV} \end{aligned}$$

	n-Si	p-Si
Au	0.885 V BAD	0.269 V REAS. GOOD
Al	0.085 V GOOD	-0.531 V BAD

BUILT-IN VOLTAGE

Good ohmic contacts:

Al - n-Si

good/Reasonable ohmic contacts:

Au - p-Si

Bad ohmic contacts:

Au - n-Si / Al - p-Si

$$\begin{aligned} \text{Si : } N_c &= 2.8 \cdot 10^{19} / \text{cm}^3 & E_g &= 1.12 \text{ eV} \\ N_v &= 1.04 \cdot 10^{19} / \text{cm}^3 & \chi &= 4.05 \text{ V} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Appendix H}$$

$$\begin{aligned} W \text{ of Au} &= 5.2 \text{ V} \\ \text{Al} &= 4.4 \text{ V} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{table / figure on p. 67}$$

of Sebenta

2)

$$W = \sqrt{\frac{2 \epsilon |V_{bi}|}{q N_A}}$$

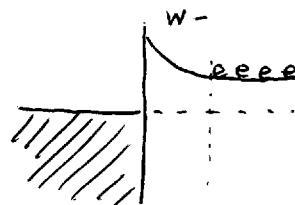
$$\epsilon = 11.9 \epsilon_0$$

$$\epsilon_0 = 8.85418 \cdot 10^{-12} \text{ F/m}$$

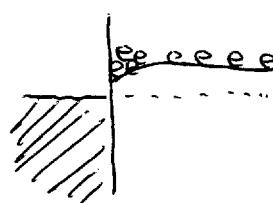
$$N_A = 10^{15} \times 10^6 \text{ m}^{-3}$$

$$C = \frac{\epsilon A}{W}$$

barrier	$ V_{bi} $	W	C
Au / n-Si	0.885 V	1078 nm	1 pF
Au / p-Si	0.269 V	accumulation!	-
Al / n-Si	0.085 V	accumulation!	-
Al / p-Si	0.531 V	836 nm	1.3 pF



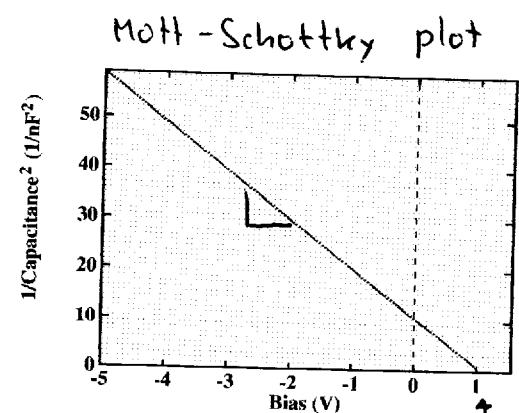
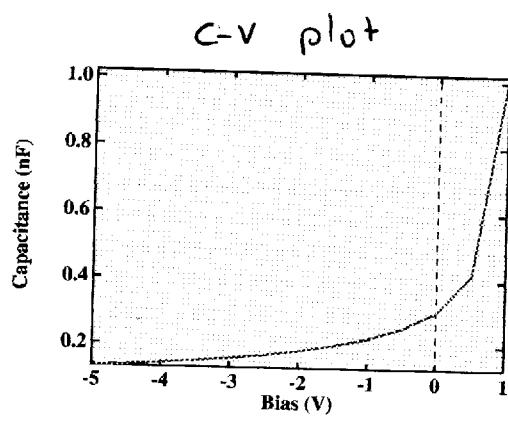
depletion



accumulation

see lectures
on MIS and
FET devices

3)



$$V_{bi} = 1.1 \text{ V}$$

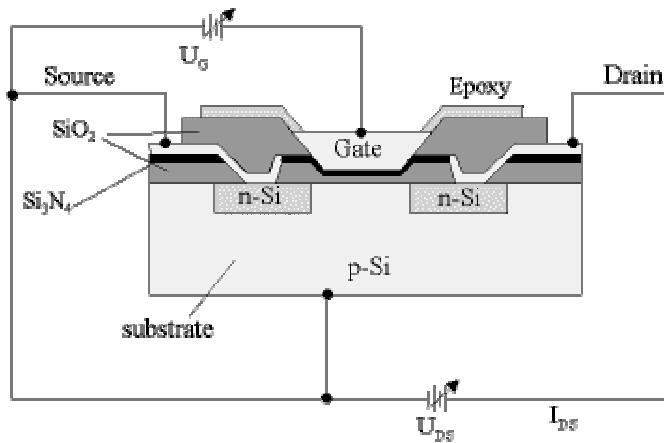
$$\frac{d \frac{1}{C^2}}{d V} = -\frac{2}{A^2 q \epsilon N_A} \Rightarrow N_A = 1.23 \cdot 10^{16} \text{ cm}^{-3}$$

FCE

Problem sheet 11

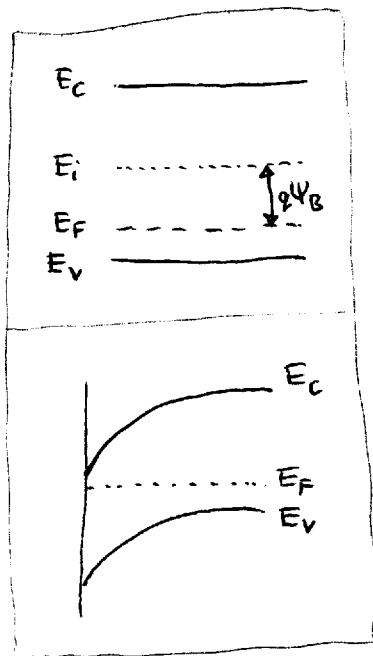
MIS diodes, FETs

- 1) Calculate the threshold voltage V_T needed to induce an inversion channel in a field-effect transistor (FET) made of p-type silicon ($N_D = 10^{17} /cm^3$). Assume an alignment of Fermi levels in the metal and the semiconductor before contact (zero band bending after contact). Oxide thickness: 200 nm. Area: 1 mm².
- 2) Where and what is the maximum electrical field in this device at this voltage?
- 3) What are the maximum and minimum capacitances of this device?
- 4) Is this larger than the breakdown field of SiO₂? (Will we burn the transistor?) What is the maximum voltage we can apply to the gate (assume $V_d = V_s = 0$)?



Solution to FCE FP II

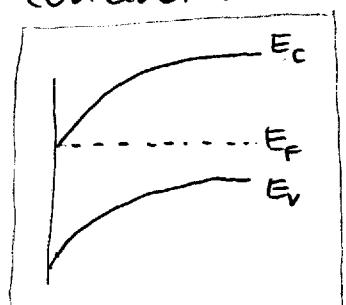
$$V_T = \frac{1}{C_{ox}} \sqrt{4q\epsilon_s \psi_B N_A} + 2\psi_B$$



With $V = V_T$ the band bending will be exactly such that, at the interface, the Fermi level depth $E_C - E_F$ is equal to $E_F - E_V$ in the bulk. In other words, the density of free electrons is equal to the density of free holes in the bulk.

The band bending is then $2\psi_B$

In the semicarta, also another band bending is used, namely $\frac{1}{2}(E_C - E_F)$: the voltage needed to put the Fermi level exactly at the bottom of the conduction band. These definitions don't differ so much and we can use either.



Substituting values: $C_{ox} = \epsilon_{ox}/d_{ox}$,

$$\epsilon_{ox} = 3.9, \epsilon_0 = 8.85418 \cdot 10^{-12} \text{ F/m}, d_{ox} = 200 \cdot 10^{-9} \text{ m}, N_A = 10^{23}/\text{m}^3, 2\psi_B \approx 1 \text{ V (bandgap)}, \epsilon_s = 11.9, q = 1.6 \cdot 10^{-9} \text{ C}$$

$$C_{ox} = 173 \text{ pF/m}^2$$

$$V_T = 11.6 \text{ V}$$

(the area $A=1 \text{ mm}^2$ doesn't enter the calculations!)

2) The maximum field is always encountered at the interface

Total voltage drop

$$\Delta V = \underbrace{\sqrt{4\epsilon_0 \epsilon_s \Psi_0 N_A} \cdot \frac{d_{ox}}{\epsilon_{ox}}}_{\text{in the oxide}} + \underbrace{2\Psi_B}_{\text{in the semiconductor}}$$

The field in the oxide is constant (because there are no charges there \Rightarrow Poisson: $\int \rho(x) dx = C$)

The field in the oxide therefore is

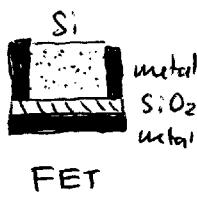
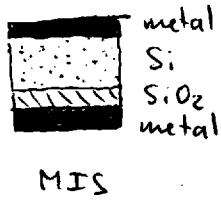
$$E_{ox} = \sqrt{4\epsilon_0 \epsilon_s \Psi_0 N_A} \frac{d_{ox}}{\epsilon_{ox}} / d_{ox}$$

$$= 10.6 \text{ V} / 200 \text{ nm}$$

$$= 5.3 \cdot 10^7 \text{ V/m}$$

4) The breakdown field of SiO_2 is (see appendix I) 10^9 V/m . We will therefore ^{not} burn this sucker.

3) We will do as if the device is an MIS diode to get an idea.



$$\text{MIS : } -||-\mid | \quad \bullet \quad \text{SiO}_2 : \quad C_{ox} = \frac{\epsilon_{ox} A}{d_{ox}} = 172 \text{ pF}$$

$$\bullet \quad \text{Si} : \quad C_{si} = \frac{\epsilon_{si} A}{W}$$

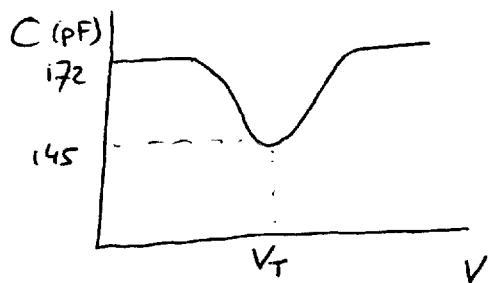
$$W=0 \quad (\text{Accumulation}) : \quad C_{si} = \infty, \quad C_{tot} = C_{ox} = 172 \text{ pF}$$

$$W_{max} : V_T, \quad V_{bb} = 2\Psi_B \approx 1 \text{ V}, \quad W = \sqrt{\frac{2\epsilon V_{bb}}{2N_A}} = 115 \text{ nm}$$

$$C = A \sqrt{\frac{\epsilon \epsilon_s N_A}{2V_{bb}}} \quad (\text{see Schottky barriers}) \\ = 919 \text{ pF}$$

(2)

$$\frac{1}{C_{min}} = \frac{1}{172 \text{ pF}} + \frac{1}{9.9 \text{ pF}} \Rightarrow C_{min} = 145 \text{ pF}$$



$$4b) \Delta V_{ox}^{\max} = E_{ox}^{\max} \cdot d_{ox} = 10^9 \text{ V/m} \cdot 200 \text{ nm} = 200 \text{ V}$$

$$\frac{1}{C_{ox}} \sqrt{2q\epsilon_s V_{bb}^{\max} N_A} = 200 \text{ V}$$

$$V_{bb}^{\max} = \frac{2^2 C_{ox}^2}{2q\epsilon_s N_A} = \frac{4 \times (173 \cdot 10^{-6})^2}{2 \times 1.6 \cdot 10^{-19} \times 11.9 \times 8.85 \cdot 10^{-12} \times N_A}$$

$$= 3.5 \text{ V}$$

$$\Delta V_{max} = \Delta V_{ox}^{\max} + V_{bb}^{\max} = 204 \text{ V}$$

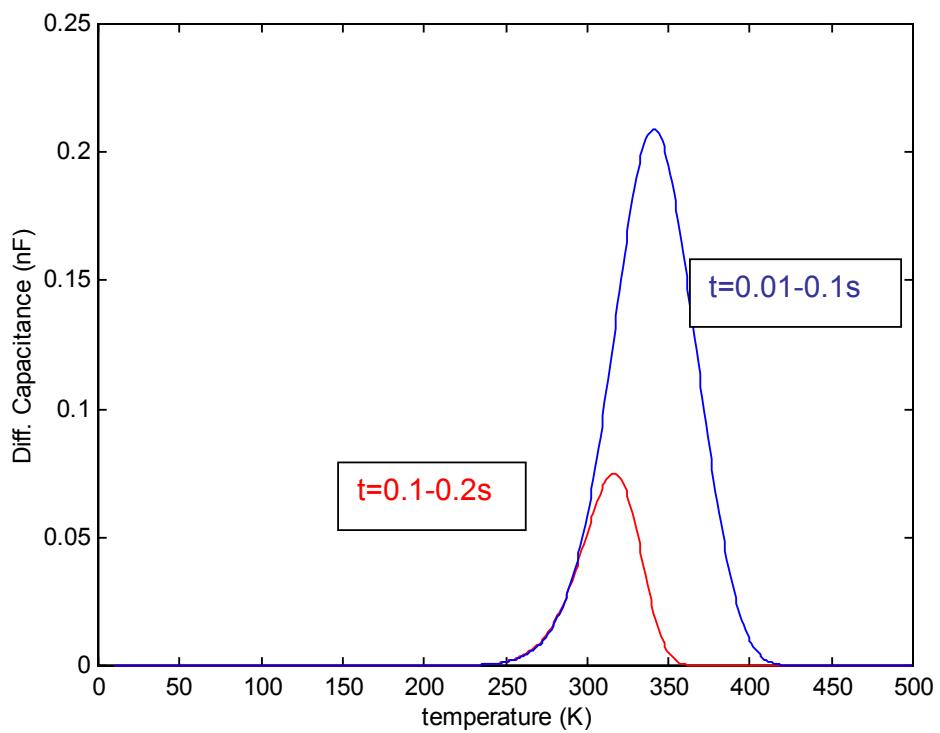
FCE

Problem sheet 12

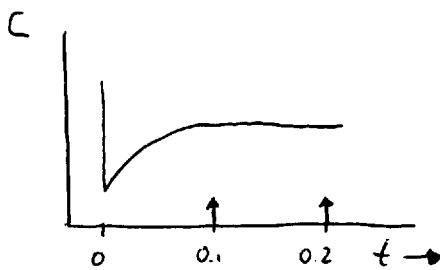
DLTS (deep-level transient spectroscopy)

- 1) Below is the result of a DLTS measurement. The first is measured with a time window of 0.1s-0.2s, while the second is with a time window of 0.01-0.1s. Determine the depth of the trap level responsible for the transients. (The files are also available on-line, in more detail).

time window 0.1s-0.2s		time window 0.01s-0.1s	
T (K)	C (nF)	T (K)	C (nF)
10.0	0.00000000000000E+0000	10.0	0.00000000000000E+0000
20.0	0.00000000000000E+0000	20.0	0.00000000000000E+0000
30.0	0.00000000000000E+0000	30.0	0.00000000000000E+0000
40.0	0.00000000000000E+0000	40.0	0.00000000000000E+0000
50.0	0.00000000000000E+0000	50.0	0.00000000000000E+0000
60.0	0.00000000000000E+0000	60.0	0.00000000000000E+0000
70.0	0.00000000000000E+0000	70.0	0.00000000000000E+0000
80.0	0.00000000000000E+0000	80.0	0.00000000000000E+0000
90.0	0.00000000000000E+0000	90.0	0.00000000000000E+0000
100.0	0.00000000000000E+0000	100.0	0.00000000000000E+0000
110.0	0.00000000000000E+0000	110.0	0.00000000000000E+0000
120.0	0.00000000000000E+0000	120.0	0.00000000000000E+0000
130.0	1.69406589450860E-0012	130.0	1.69406589450860E-0012
140.0	3.72694496791892E-0011	140.0	3.30342849429177E-0011
150.0	5.12454933088852E-0010	150.0	4.61632956253594E-0010
160.0	5.14064295688635E-0009	160.0	4.62649395790299E-0009
170.0	3.95776144604572E-0008	170.0	3.56194294979378E-0008
180.0	2.44478272533061E-0007	180.0	2.20030360576461E-0007
190.0	1.25408733555156E-0006	190.0	1.12868241364467E-0006
200.0	5.49143995792582E-0006	200.0	4.94238219008727E-0006
210.0	2.09897161432856E-0005	210.0	1.88920000858948E-0005
220.0	7.13166140793348E-0005	220.0	6.41994544683814E-0005
230.0	2.18615840118501E-0004	230.0	1.96890662207774E-0004
240.0	6.11845668158355E-0004	240.0	5.51732314180056E-0004
250.0	1.57747517288688E-0003	250.0	1.42689398470940E-0003
260.0	3.76748962549570E-0003	260.0	3.43222484209881E-0003
270.0	8.34303617961867E-0003	270.0	7.71877163033096E-0003
280.0	1.70323112201837E-0002	280.0	1.62636542501735E-0002
290.0	3.15513572748930E-0002	290.0	3.20392365194111E-0002
300.0	5.14296433563570E-0002	300.0	5.85593597902964E-0002
310.0	7.00472388832874E-0002	310.0	9.78390155181765E-0002
320.0	7.36402353526528E-0002	320.0	1.46088250120050E-0001
330.0	5.34580267287079E-0002	330.0	1.89614803198390E-0001
340.0	2.32318703086620E-0002	340.0	2.08966493080889E-0001
350.0	5.08986530189709E-0003	350.0	1.94731438325217E-0001
360.0	4.50496216191005E-0004	360.0	1.56167522562089E-0001
370.0	1.15896079219040E-0005	370.0	1.08585373163245E-0001
380.0	5.35426466618388E-0008	380.0	6.34275959967396E-0002
390.0	2.37169225231204E-0011	390.0	2.92429728554407E-0002
400.0	0.00000000000000E+0000	400.0	9.78652818065830E-0003
410.0	0.00000000000000E+0000	410.0	2.13698165032453E-0003
420.0	0.00000000000000E+0000	420.0	2.66385853059750E-0004
430.0	0.00000000000000E+0000	430.0	1.60689976678573E-0005
440.0	0.00000000000000E+0000	440.0	3.83302737423305E-0007
450.0	0.00000000000000E+0000	450.0	2.83417224151289E-0009
460.0	0.00000000000000E+0000	460.0	5.08219768352580E-0012
470.0	0.00000000000000E+0000	470.0	0.00000000000000E+0000
480.0	0.00000000000000E+0000	480.0	0.00000000000000E+0000
490.0	0.00000000000000E+0000	490.0	0.00000000000000E+0000

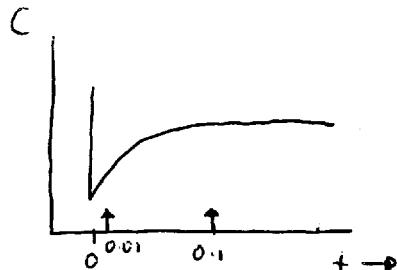


Solution to FCE FP 12



time window
0.1 s - 0.2 s

$$T_{\max} = 316 \text{ K}$$



time window
0.01 s - 0.1 s

$$T_{\max} = 341 \text{ K}$$

$$\begin{aligned} T_{\max} &= (t_2 - t_1) / \ln\left(\frac{t_2}{t_1}\right) \\ &= 144.3 \text{ ms} \end{aligned}$$

$$\begin{aligned} T_{\max} &= (t_2 - t_1) / \ln\left(\frac{t_2}{t_1}\right) \\ &= 43.0 \text{ ms} \end{aligned}$$

p. 81 of
Sebenta

$$\tau_{\max} = T_0 T_{\max}^2 \exp\left(\frac{E_a}{k T_{\max}}\right)$$

$$\frac{T_{\max,2}}{T_{\max,1}} = \left(\frac{T_{\max,2}}{T_{\max,1}}\right)^2 \exp\left(\frac{E_a}{k} \left[\frac{1}{T_{\max,2}} - \frac{1}{T_{\max,1}}\right]\right)$$

$$\begin{aligned} E_a &= \frac{k \left[\ln\left(\frac{T_{\max,2}}{T_{\max,1}}\right) - 2 \ln\left(\frac{T_{\max,2}}{T_{\max,1}}\right) \right]}{\frac{1}{T_{\max,2}} - \frac{1}{T_{\max,1}}} \end{aligned}$$

$$= 0.51 \text{ eV}$$

What is the error margin in this value?

- The T_{\max} can be determined with 1 K precision

Assuming the error only comes from determining T_{\max} and the curves are noise-less transients.

$\Delta T = 1 \text{ K}$. We have 2 T's

Total error

$$\Delta T = \sqrt{\Delta T_{\max,1}^2 + \Delta T_{\max,2}^2} = 1.41 \text{ K}$$

Now substituting

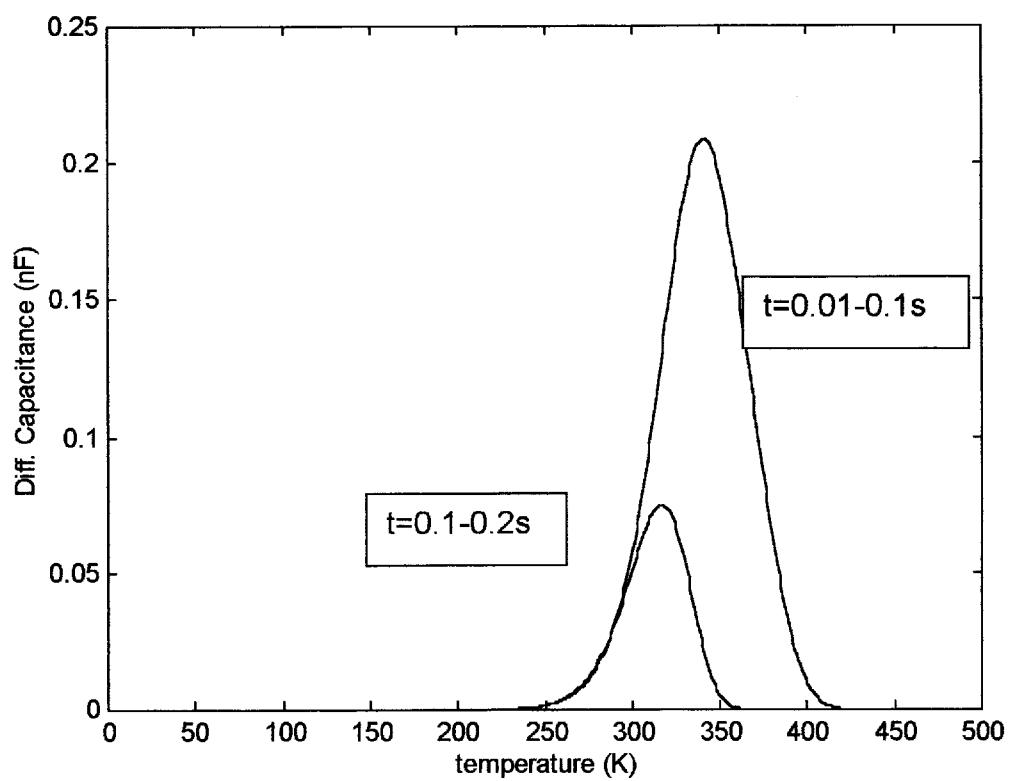
$$T_{\max,1} \rightarrow T_{\max,1} + 1.41 \text{ K}$$

$$316 \text{ K} \rightarrow 317.4 \text{ K}$$

This gives

$$E_a = 0.54 \text{ eV}$$

The error is $\pm 0.03 \text{ eV}$



FCE

Extra problem sheet

Anomalies in FET curves

In FET curves we can sometimes encounter non-linearities. One example is the effect created by the electrodes. So far we have assumed that the source and drain electrodes easily inject holes or electrons into the channel of the FET. With our knowledge of Schottky barriers (contacts of metals with semiconductors) we know that normally metals form rectifying contacts with semiconductors. We can simulate this by putting a diode in series with the FET.

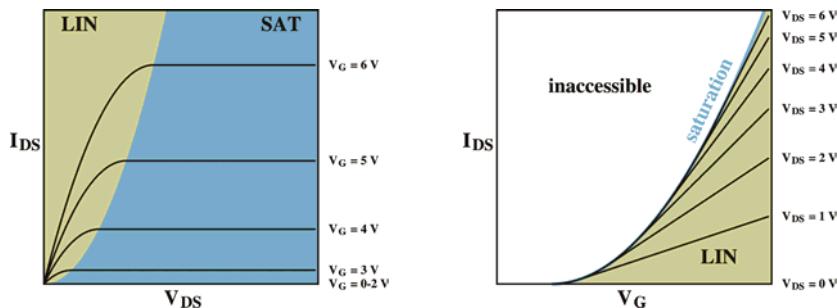
1. Make a simulation (with a program like Spice, Electronic Workbench, or Matlab) of an FET suffering from Schottky barrier contacts.

To circumvent these effects, we can make a highly-doped region at the contacts, thereby making the contacts ohmic (with only a resistance and no rectifying effects), as described in the *sebenta*.

2. Make a simulation of an FET with ohmic contacts at the source and the drain.

In both cases make simulations of the IV curves (I_{ds} vs. V_{ds}) and transfer curves (I_{ds} vs. V_g) and use values for the parameters (diode parameters and resistance value) that clearly show what is going on. Make a summary with the conclusions.

3. Find out in literature what is “velocity saturation”. Describe this in a report with figures, etc.
4. Find out in literature what are “short-channel effects”. Describe this in a similar way.
5. Describe, preferably with a simulation, what will happen if the mobility depends on the in-plane field (V_{ds}) and the transverse field (V_g).



Teste de Fundamentos de Componentes Electrónicos

16 de Junho 2003, 15:00-17:00 (Duração: 2 horas)
Universidade do Algarve



-
- Escreva o seu nome, número e curso em todas as folhas que entregares.
 - Não é permitido falar com os colegas durante o exame. Se o fizer, a sua prova será anulada. Desligue o telemóvel.
 - Caso opter por desistir, escreva “Desisto”, assine e entregue a prova ao docente.
 - O exame tem 5 perguntas e a cotação de cada aparece entre parêntesis.
 - Use letra legível.
 - Sempre mostre os **cálculos** que resultaram a chegar às respostas. Onde possível, também use **gráficos** para explicar as coisas.
 - Boa sorte!
-

Pergunta 1 (2 valores)

Distinga um semicondutor de um metal.

Pergunta 2 (11 valores)

- Determine a posição do nível de Fermi e as densidades de portadores (electrões e lacunas) num cristal puro de silício à temperatura de 300 K.
- Repite a alínea anterior com uma dopagem de 10^{16} cm^{-3} de átomos de Indium (In).
- Explique o que acontecerá quando variar a temperatura numa gama grande (por exemplo 1 K - 2000 K).
- Numa fábrica, alguém sugeriu usar Indium para fabricar as pistas dos dispositivos, porque assim o custo da produção seria mais baixo. Desenhe o diagrama de bandas e calcule os parâmetros do dispositivo (*barrier height* e *built-in voltage*).
- Acha que os contactos eléctricos com o dispositivo tem capacidade? Se sim, calcule o valor da capacidade (use uma área de contacto de 1 mm^2)?
- O Indium será um bom metal para fabricar as píscas de ligação aos dispositivos? Porquê?
- Pode sugerir formas de minimizar os efeitos devido aos contactos.

Pergunta 3 (2 valores)

Explique porque o diamante é transparente e o silício tem uma cor cinzenta. Sendo ambos materiais semicondutores. Pode confirmar a sua resposta com valores numéricos.

Pergunta 4 (3 valores)

A técnica de correntes termoestimuladas conhecida vulgarmente por TSC (Temperature Stimulated Current) permite identificar níveis de impurezas profundas em materiais semicondutores. Explique como funciona a técnica (não precisa de realizar cálculos).

Pergunta 5 (2 valores).

Explique como o átomo de silício pode ser uma impureza dadora e também uma impureza aceitadora no arseneto de gálio.

Exame de Fundamentos de Componentes Electrónicos

4 de Julho 2003, 9:30-11:30 (Duração: 2 horas)

Universidade do Algarve



-
- Escreva o seu nome, número e curso em todas as folhas que entregares.
 - Não é permitido falar com os colegas durante o exame. Se o fizer, a sua prova será anulada. Desligue o telemóvel.
 - Caso opte por desistir, escreva “Desisto”, assine e entregue a prova ao docente.
 - O exame tem 4 perguntas e a cotação de cada aparece entre parêntesis.
 - Use letra legível.
 - Sempre mostre os **cálculos** que resultaram a chegar às respostas. Onde possível, também use **gráficos** para explicar as coisas.
 - Boa sorte!
-

Pergunta 1 (3 valores)

Explique o conceito de massa efectiva. Dê um exemplo numérico de SiC.

Pergunta 2 (11 valores)

O assunto de esta pergunta é “díodos do tipo MIS”. Imagine um díodo MIS de GaAs dopado com $10^{15} / \text{cm}^3$ de átomos de Pb e o metal Pb.

- a) O que é um díodo MIS? Porque um díodo ideal MIS não conduz corrente?
- b) Senão conduz corrente, qual é a utilidade de um díodo tipo MIS?
- c) Determine o nível de Fermi no lado do GaAs ($T = 300 \text{ K}$).
- d) Qual será o *bandbending* e a largura de zona de depleção na parte de GaAs se não tiver uma tensão externa ligada (espessura da camada isoladora: 200 nm).
- e) Neste ponto de funcionamento, o dispositivo está a funcionar em qual regime?
- f) Qual será a tensão necessária para induzir um canal do tipo inversão e qual será a tensão necessária para induzir um canal do tipo acumulação?
- g) Para o último caso (de canal do tipo acumulação), mostre
 - O diagrama energético (Energia em função de espaço)
 - O diagrama de carga espacial (*space charge*; Carga total em função de espaço)
 - O diagrama de carga livre (Carga livre em função de espaço)
 - Dê nomes às várias áreas do dispositivo.

Pergunta 3 (3 valores)

A técnica conhecida vulgarmente por DLTS (Deep-Level Transient Spectroscopy) permite identificar níveis de impurezas profundas em materiais semicondutores. Explique como funciona a técnica (não precisa de realizar cálculos).

Pergunta 4 (3 valores).

Uma das vantagens de silício para a sua aplicação na indústria electrónica é que existe um bom isolador (SiO_2) que é fácil a produzir (só é necessário oxidar a bolacha de silício em ar a uma temperatura aproximadamente $800 \text{ }^\circ\text{C}$) e que permite fabricar “chips”, electrónica integrada com muitos componentes isolados. Para os outros materiais, produzir um isolador torna-se mais difícil, até ao ponto que a qualidade dos dispositivos é muito inferior aos feitos de silício, por enquanto, teóricamente deveria ser superior (maior mobilidade, etc).

Um método usado para produzir camadas isoladoras em GaAs é implantar excessivamente átomos de As. Alguns de estes átomos vão substituir átomos de Ga,

tornando-se assim *double-donor* profundos (com dois níveis *mid-gap*, no meio do hiato).

Do outro lado, *Fermi-level pinning* é fixar o nível de Fermi em cima de, ou perto de um determinado nível.

- Explique porque o As será um double-donor em GaAs
- Explique (preferencialmente com diagramas, figuras, plots, imagens) como é que implantar muitos átomos de As no GaAs ligeiramente dopado com aceitadores vai causar um *Fermi level pinning* perto do meio do hiato de energia e como é que isto torne o GaAs isolador.

Exame de Recurso de Fundamentos de Componentes Electrónicos

14 de Julho 2003, 9:30-11:30 (Duração: 2 horas)

Universidade do Algarve



-
- Escreva o seu nome, número e curso em todas as folhas que entregares.
 - Não é permitido falar com os colegas durante o exame. Se o fizer, a sua prova será anulada. Desligue o telemóvel.
 - Caso opte por desistir, escreva “Desisto”, assine e entregue a prova ao docente.
 - O exame tem 5 perguntas e a cotação de cada aparece entre parêntesis.
 - Use letra legível.
 - Sempre mostre os **cálculos** que resultaram a chegar às respostas. Onde possível, também use **gráficos** para explicar as coisas.
 - Boa sorte!
-

Pergunta 1 (4 valores)

- Desenhe a estrutura de um cristal de silício (ou diamante).
- Qual é o nome de este estrutura?
- Em base da estrutura acima, calcule a densidade de silício. Use os parâmetros dados nas tabelas.
- Dê exemplos de defeitos intrínsecos de silício.

Pergunta 2 (4 valores)

Para qual temperatura um nível de cobalt (Co, $N_{Co} = 3 \times 10^{16} \text{ cm}^{-3}$) será meio cheio / meio vazio em GaAs?

Pergunta 3 (4 valores)

A técnica de Hall consegue determinar a mobilidade das cargas livres. Explique como funciona a técnica (não precisa realizar cálculos).

Pergunta 4 (4 valores).

Prove que a tensão de arranque de um transistor tipo MOSFET é dado pela equação

$$V_T = \frac{\sqrt{(4 q \varepsilon_s \Psi_B N_A)}}{C_{ox}} + 2 \Psi_B$$

q : unidade elementária de carga, N_A : densidade de aceitadores, C_{ox} : densidade de capacidade do isolador, Ψ_B : a distância do nível de Fermi ao nível de Fermi intrínseco.

Pergunta 5 (4 valores).

- Desenhe as curvas de um FET (curva de transferência e curva de conductância).
- Como determinar a mobilidade em base das curvas?

Normalmente assume-se que a mobilidade das cargas livres é constante e só depende da temperatura. Nos alguns materiais, a mobilidade depende do campo eléctrico E da forma

$$\mu = \mu_0 \exp(E/kT)$$

- Qual seria o efeito nas formas das curvas acima?

C	$C = \varepsilon A/d$
$f(E)$	distribuição Fermi-Dirac: $f(E) = \frac{1}{1 + \text{Exp}[(E-E_f)/kT]}$
	Equação Poisson: $\frac{d^2V}{dx^2} = \rho(x)/\varepsilon$
$E(x)$	campo eléctrico $E(x) = dV/dx$
E_k	energia cinética $E_k = \frac{1}{2} mv^2$ (física clássica), $E_k = \hbar^2 k^2 / 2m_e$ (física quântica)
k	wavenumber (número de onda) $k = 2\pi/\lambda$
p	momento $p = mv$ (física clássica), $p = \hbar k / 2\pi$ (física quântica)
λ	comprimento de onda $\lambda = c/v$ (v é frequência) Relação de De Broglie $\lambda = h/p$
	Equação de Einstein $E = mc^2$
E	energia de um fotão $E = h\nu$
E_H	energia de um electrão a volta de um protão (modelo hidrogénio) $E_H = \frac{mq^4}{32\pi\hbar^2\varepsilon_r^2\varepsilon_0^2}$
J_n	current density equation (equação de densidade de corrente) $J_n = q \mu_n n E_x + q D_n dn/dx$
J_n	$J_n = \mu_n n \frac{dE_F}{dx}$
J	Equação Shockley $J(V) = J_s [\exp(qV/kT) - 1]$ $J_s = q n_0 \sqrt{D_n/\tau} + q p_0 \sqrt{D_p/\tau}$
W	largura de <u>zona de depleção</u> $W = \sqrt{\frac{2 \varepsilon (V_{bi} - V)}{q N_A}}$
e_p	taxa de emissão $e_p = \gamma T^2 \sigma_{pa} \exp \left(- \frac{E_a}{kT} \right)$ $\tau = 1/e_p$
V_T	tensão de arranque $V_T = \frac{\sqrt{4 q \varepsilon_s \Psi_B N_A}}{C_{ox}} + 2 \Psi_B$ $\Psi_B = (E_i - E_f)/q$