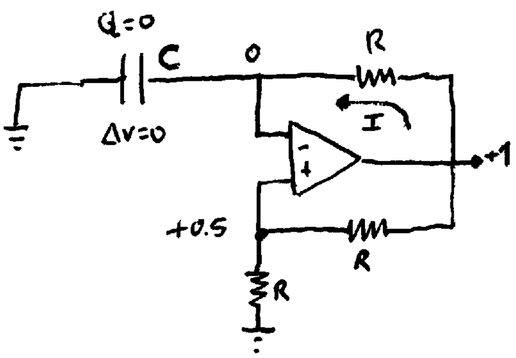
Electronics II Problem sheet 7

Initial situation, just after switching on circuit:



Capacitor:

$$C \equiv \frac{Q}{\Delta V}$$

$$\Delta V = \frac{Q}{C} = \frac{\int I(t)\,dt}{C}$$

NB: all voltages relative (in units of) $+V_{cc}$: $V_{cc} \to 1$, etc.

Resistance at negative feedback loop (R, C) feels a voltage drop of $1 - 0 = 1$. This induces a current (Ohm's Law) of $I = \frac{1}{R}$ This current cannot enter the op-amp ($r_{in} = \infty$), therefore can only charge C. Q of C will increase, making ΔV larger. This will decrease the voltage drop accross R. Thus we recognize a relaxation system

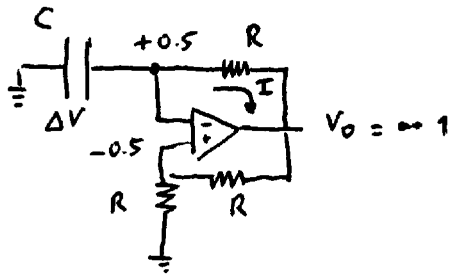
$$V_n(t) = 1 - \exp(-t/\tau), \quad \tau = RC$$

[check: $V_n(0) = 0$, $V_n(\infty) = 1$] [note: $I(t=\infty) = 0$]

V_n will never reach $+1$, because at $V_n = +0.5$ the voltage is bigger than V_p and the comparator op-amp will commutate $V_0 \to -1$ (and $V_p \to -0.5$). At this moment a cycle starts and we have the following situation

$V_0 = -1$, $V_p = -0.5$, $V_n = +0.5$.

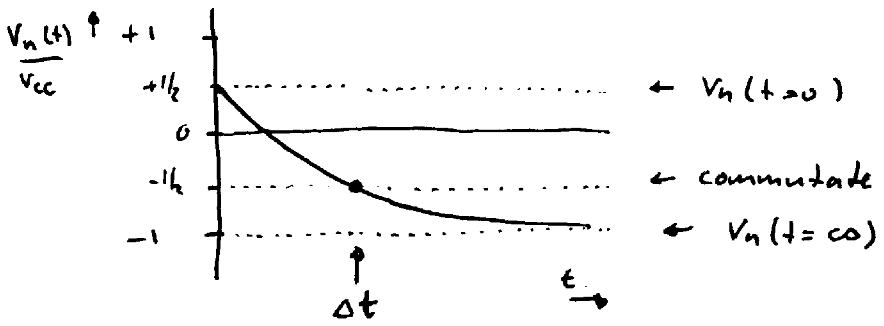
Note that capacitors keep their ΔV's because Q cannot change instantaneously.



The resistance at the negative feedback loop feels a voltage drop of 1.5. This induces a current that will change the Q in C and thus change ΔV, the current will drop. This is a relaxation system. $V_n(t=0) = +0.5$, $V_n(t=\infty) = -1$ ($I(t=\infty)=0$), and the relaxation time is RC:

$$V_n(t) = -1 + 1.5 \exp(-t/\tau)$$

V_n never reaches -1, because at $V_n = -0.5$ it will drop below V_p and the comparator will commutate $-1 \rightarrow +1$.

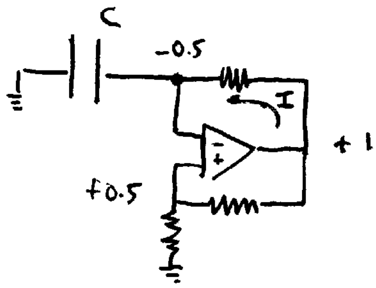


At $t = \Delta t$: $V_n(t) = -0.5$

$$-1 + 1.5 \exp(-t/\tau) = -0.5$$

$$\Rightarrow \quad t = \tau \ln(3)$$

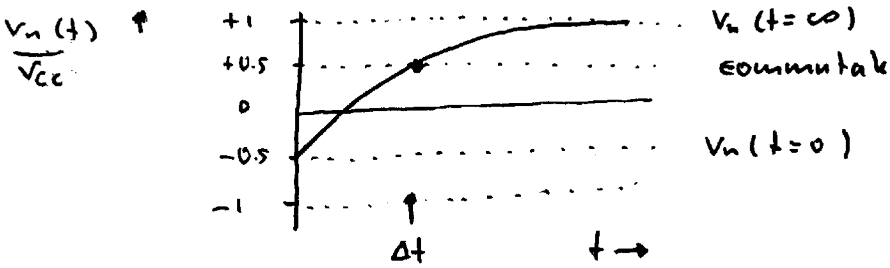
Now a second half cycle starts



Relaxation system with $V_n(t=0) = -0.5$, $V_n(t=\infty) = +1$ $(I(t=\infty)=0)$, and $\tau = RC$

$$V_n(t) = 1 - 3/2 \exp(-t/\tau)$$

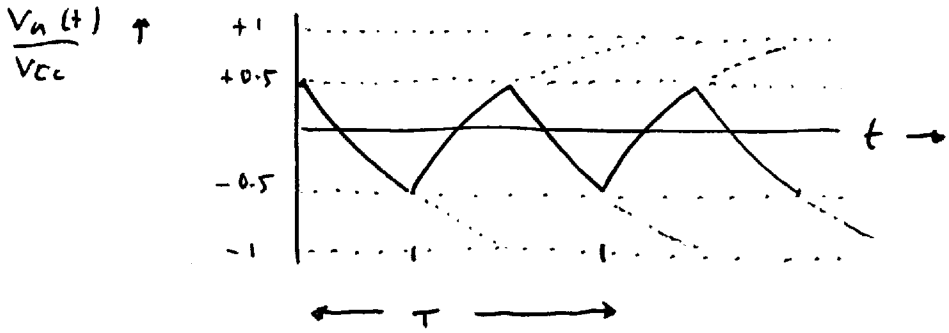
V_n never reaches $+1$, because at $V_n = +0.5$, V_o will commutate



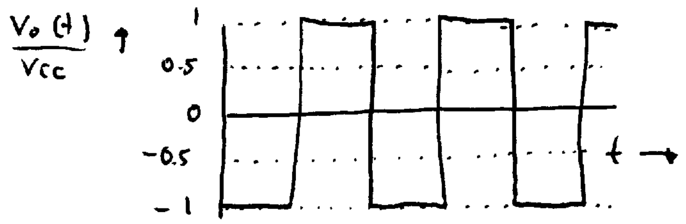
At $t = \Delta t$: $V_n(t) = +0.5$

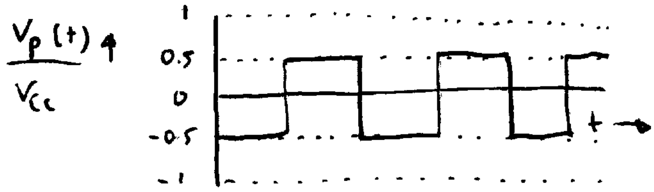
$$1 - 3/2 \exp(-t/\tau) = 1/2$$

$$\Rightarrow t = \tau \ln(3).$$



$$T = 2RC\ln(3) \Rightarrow f = \frac{1}{2RC\ln(3)}$$





$$A(s) = \left(\frac{10}{1 + s/10^4}\right)^3, \qquad A(\omega) = \left(\frac{10}{1 + j\omega/10^4}\right)^3$$

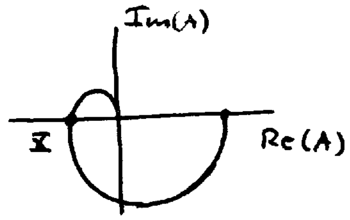
$$\left.\begin{array}{lll} \omega = 0 & \Rightarrow A(0) = & 1000 \\ \omega = \infty & \Rightarrow A(\infty) = & 0 \end{array}\right\} \text{LPF}$$

three poles at $\omega = 10^4$

Every pole introduces a phase shift of $-90°$

phase at $\omega = 0$: $0°$

$\Rightarrow$ phase at $\omega = \infty$: $-270°$



Nyquist plot of A

At point X phase $= -180°$

This means $\text{Im}(A) = 0$

$$A(\omega) = \frac{10^3}{1 + \frac{3j\omega}{10^4} + \frac{3j^2\omega^2}{(10^4)^2} + \frac{j^3\omega^3}{(10^4)^3}} = \frac{10^3}{\left(1 - \frac{3\omega^2}{10^8}\right) + j\left(\frac{3\omega}{10^4} - \frac{\omega^3}{10^{12}}\right)}$$

$$\text{Im}(A(\omega)) = 0 \Rightarrow \frac{3\omega}{10^4} - \frac{\omega^3}{10^{12}} = 0$$

$$\Rightarrow \omega = 0, \text{ or } \omega = 10^4\sqrt{3}$$

At this frequency, $\omega_o = 10^4 \times \sqrt{3}$

$$|A(\omega_o)| = \left| \frac{10^3}{1 - 3\,\omega_o^2/10^8} \right| = \frac{10^3}{-1 + 3 \times (\sqrt{3} \times 10^4)^2/(10^4)^2} = \frac{10^3}{8}$$

Stable if $|A\beta| < 1$

$$|\beta| < 1/|A| = \frac{8}{10^3} = 0.008$$

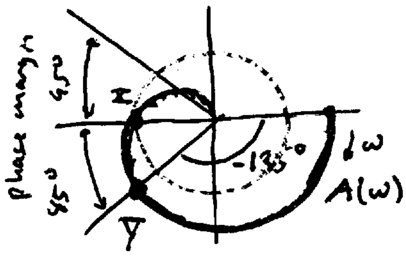
No, this is not correct!

$\uparrow$ $\beta_{cr.}$

At every pole, $\Delta\theta = -45°$

Three times same pole: at this frequency $\Delta\theta = 3 \times (-45°) =$

$-135°$. This is exactly our phase margin $(-180° \pm 45°)$



$\omega_p = 10^4$ rad/s

$$Y: A(\omega_p) = \left(\frac{10}{1 + j\frac{10^4}{10^4}} \right)^3 = \left(\frac{10}{1+j} \right)^3 = \frac{10^3}{(-2 + 2j)}$$

$$|A(\omega_p)| = \frac{10^3}{2\sqrt{2}} = 353.6$$

Stable if at this phase $|A(\omega_p)\,\beta(\omega_p)| \leq 1$

$$\beta \leq \frac{1}{|A(\omega_p)|} = \frac{2\sqrt{2}}{10^3} = 2.83\ 10^{-3}$$

$\uparrow$ β_{cr}

Phase margin $\neq 45°$

$$A(\omega) = \frac{10^3}{\left[1 - 3\left(\frac{\omega}{10^4}\right)^2\right] + j\left[3\left(\frac{\omega}{10^4}\right) - \left(\frac{\omega}{10^4}\right)^3\right]}$$

$$= \frac{10^3}{\left[1 - 3\left(\frac{\omega}{10^4}\right)^2\right] + j\left[3\left(\frac{\omega}{10^4}\right) - \left(\frac{\omega}{10^4}\right)^3\right]} \times \frac{\left[1 - 3\left(\frac{\omega}{10^4}\right)^2\right] - j\left[3\left(\frac{\omega}{10^4}\right) - \left(\frac{\omega}{10^4}\right)^3\right]}{\left[1 - 3\left(\frac{\omega}{10^4}\right)^2\right] - j\left[3\left(\frac{\omega}{10^4}\right) - \left(\frac{\omega}{10^4}\right)^3\right]}$$

$$tg(\theta) = \frac{Im(A)}{Re(A)} = \frac{-3\left(\frac{\omega}{10^4}\right) + \left(\frac{\omega}{10^4}\right)^3}{1 - 3\left(\frac{\omega}{10^4}\right)^2} = tg(-180° + P.M.)$$

trivial case : P.M. $= 45°$ $\rightarrow$ $tg(-180° + 45°) = 1$

$\Rightarrow \omega = 10^4$, $A(10^4) = \dfrac{10^3}{-2 + 2j}$ ... etc.

non-trivial case, ex. P.M. $= 30°$ $\rightarrow$ $tg(-180° + 30°) = \dfrac{1}{\sqrt{3}}$

$$\rightarrow \frac{x^3 - 3x}{1 - 3x^2} = \frac{1}{\sqrt{3}} \qquad x = \left(\frac{\omega}{10^4}\right)$$

$\rightarrow x = 1.1918$ $\Rightarrow$ $\omega = 1.1918 \; 10^4$ rad/s $\equiv \omega_x$

(solved with Octave)

$$|A(\omega_x)| = \frac{10^3}{\sqrt{\left[1 - 3\left(\frac{\omega_x}{10^4}\right)^2\right]^2 + \left[3\left(\frac{\omega}{10^4}\right) - \left(\frac{\omega}{10^4}\right)^3\right]^2}} = 265.58$$

$$|A(\omega_x)| \cdot |\beta| \leq 1 \Rightarrow \beta = \frac{1}{0.26558 \times 10^3} = 0.0037653$$

$\uparrow$

β_{cr}