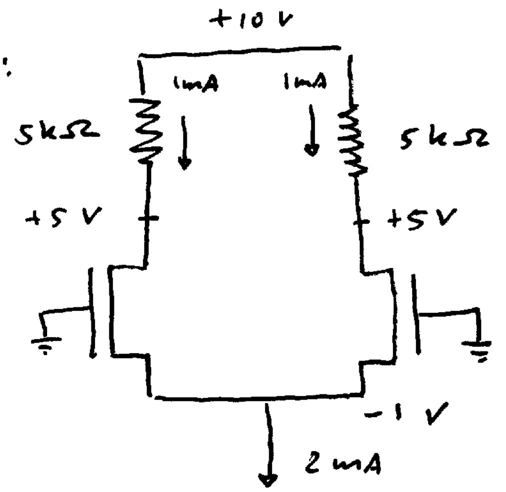
Folha de problemas 3

1:

a) Bias:



$V_T = 0$

$K = 2\,mA/V^2$

SAT: $I_{D1} = \frac{K}{2} V_{GS}^2 = I_{D2} = 1\,mA \Rightarrow$

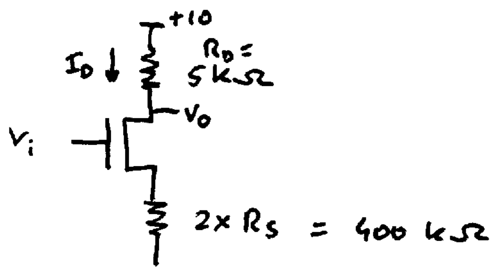
$$\frac{2\,mA/V^2}{2} \cdot (0 - V_S)^2 = 1\,mA \Rightarrow$$

$$V_S = -1\,V$$

(check if in saturation : $V_{DS} > V_{GS} - V_T$: $6 > 1$ ok!)

b) Common mode gain:

Equivalent circuit . use technique of symmetry



$$A_V = \frac{dV_o}{dV_i} = -\frac{dI_D}{dV_G} \times R_D$$

$$I_D = \frac{k}{2}(V_G - V_S)^2 \quad . \quad V_S = -1\,V + (I_D - 1\,mA) \cdot 2R_S$$

$$I_D = \frac{k}{2}\left(V_G + 1V - (I_D - 1mA) \times 2R_S\right)^2$$

$$V_G = \mp 1V + (\cancel{1mA - I_D})^{I_D - 1mA} \times 2R_S + \sqrt{\frac{2}{k}}\sqrt{I_D}$$

$$\frac{dV_G}{dI_D} = +2R_S - \sqrt{\frac{2}{K}} \cdot \frac{1}{2\sqrt{I_D}} = +400\,k\Omega - 500\,\Omega$$

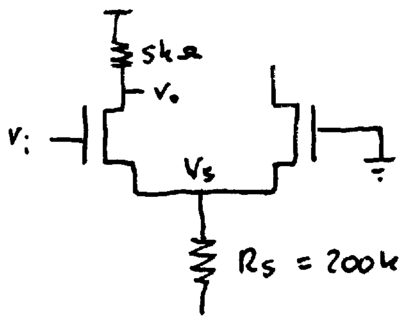
$$= 399.5\,k\Omega$$

$$\frac{dI_D}{dV_G} = 1/\frac{dV_G}{dI_D} = \frac{1}{399.5\,k\Omega} \approx 2.5\,\mu S$$

$$A_v^{cm} = -2.5\,\mu S \times 5\,k\Omega = -0.0125$$

common mode gain $= \boxed{-0.0125}$

differential mode gain :



Assumption : $I_{D1} + I_{D2} = I_s$

$= \text{constant} = 2\,mA$

$\Rightarrow V_S = -1\,V + \frac{V_i}{2}$

$$I_{D1} = \frac{K}{2}(V_{G1} - V_S)^2$$

$$= \frac{K}{2}\left(V_i + 1\,V - \frac{V_i}{2}\right)^2 = \frac{K}{2}(V_i/2 + 1\,V)^2$$

$$g_m = \frac{dI_{D1}}{dV_i} = 2\frac{K}{2}\left(\frac{V_i}{2} + 1\,V\right)\cdot\frac{1}{2} = \frac{K}{2}\left(\frac{V_i}{2} + 1\,V\right)$$

small signal $V_i \approx 0 \Rightarrow g_m = \frac{K}{2} \times 1\,V$

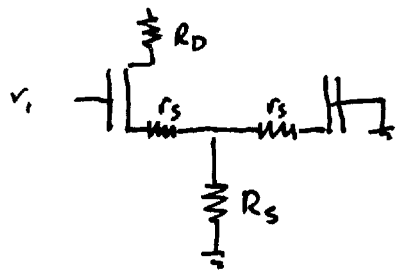
$$= \frac{2\,mA/V^2}{2} \times 1\,V = 1\,mS$$

$$A_v = \frac{dV_o}{dV_i} = -\frac{dI_{D1}}{dV_i} \times R_D = -g_m R_D = -1\,mS \times 5\,k\Omega = \boxed{-5}$$

$$\boxed{CMRR = \left|\frac{-5}{-0.0125}\right| = 400}$$

Note : we can use knowledge of electronics I to find a faster answer :

The gain of common - source amplifier is given by : all resistance at drain divided by all resistance at source with a minus sign.



The r_s resistances are <u>modelling</u> resistances equivalent to r_e for bipolar transistor. They are virtual resistances that describe behavior of I_S (I_E). For bipolar r_e was found to be $\left(\frac{\beta+1}{\beta} g_m\right)^{-1}$

$$r_e \equiv 1 \Big/ \frac{\partial I_E}{\partial v_{BE}} = 1 \Big/ \frac{\beta+1}{\beta} \cdot \frac{d I_C}{d v_{BE}} = 1 \Big/ \frac{\beta+1}{\beta} \cdot g_m$$

For a FET

$$r_s \equiv 1 \Big/ \frac{\partial I_S}{\partial v_{GS}} = 1 \Big/ \frac{\partial I_D}{\partial v_{GS}} = 1/g_m$$

$$g_m = \frac{dI_D}{dV_{GS}} = KV_{GS} = \frac{K/2\, V_{GS}^2}{1/2\, V_{GS}} = \frac{2 I_D}{V_{GS}}$$

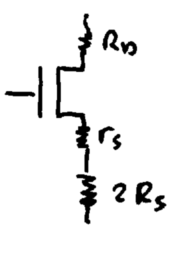
The gain of the common - source configuration above is thus

$$\frac{v_o}{v_i} = - \frac{R_D}{r_s + r_s /\!/ R_s} \approx -\frac{R_D}{2 r_s}$$

$$r_s = 1/g_m$$
$$g_m = \frac{2 I_D}{V_{GS}} = \frac{2\text{ mA}}{1\text{ V}} = 2\text{ mS}$$
$$r_s = 500\ \Omega$$

$$= -\frac{5000}{1000} = -5$$

The common mode common source configuration:



$$\frac{v_o}{v_i} = - \frac{R_D}{r_s + 2R_s} \approx - \frac{5000}{400\,000} = -0.0125$$

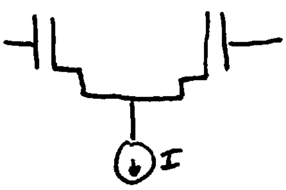
c) $r_{in} \equiv 1 / \frac{\partial I_{in}}{\partial v_{in}}$. $I_{in} = I_G = 0 \Rightarrow r_{in} = \infty$

$r_{out} \equiv 1 / \frac{\partial I_{out}}{\partial V_{out}} = R_D // r_o$. $r_o = \infty \Rightarrow r_{out} = R_D$

of FET

---

2 : For a differential pair based on bipolar transistors:
See page 490 of Sedra (Large - Signal operation of the BJT Diferential Pair)
For a differential pair based on FETS :
See page 527 of Sedra (MOS differential amplifiers)



$$I_{D1} = \frac{K}{2} (V_{GS1} - V_T)^2$$

$$I_{D2} = \frac{K}{2} (V_{GS2} - V_T)^2$$

$$\sqrt{I_{D1}} = \sqrt{\frac{K}{2}} (V_{GS1} - V_T) \quad , \quad \sqrt{I_{D2}} = \sqrt{\frac{K}{2}} (V_{GS2} - V_T)$$

$$\sqrt{I_{D1}} - \sqrt{I_{D2}} = \sqrt{\frac{K}{2}} (V_{GS1} - V_{GS2}) = \sqrt{\frac{K}{2}}\, v_i \qquad (A)$$

$$I_{D1} + I_{D2} = I \qquad (B)$$

combine (A) and (B) :

$$I_{D1} = \frac{I}{2} + \sqrt{KI}\, \frac{v_i}{2} \sqrt{1 - \frac{(v_i/2)^2}{I/K}}$$

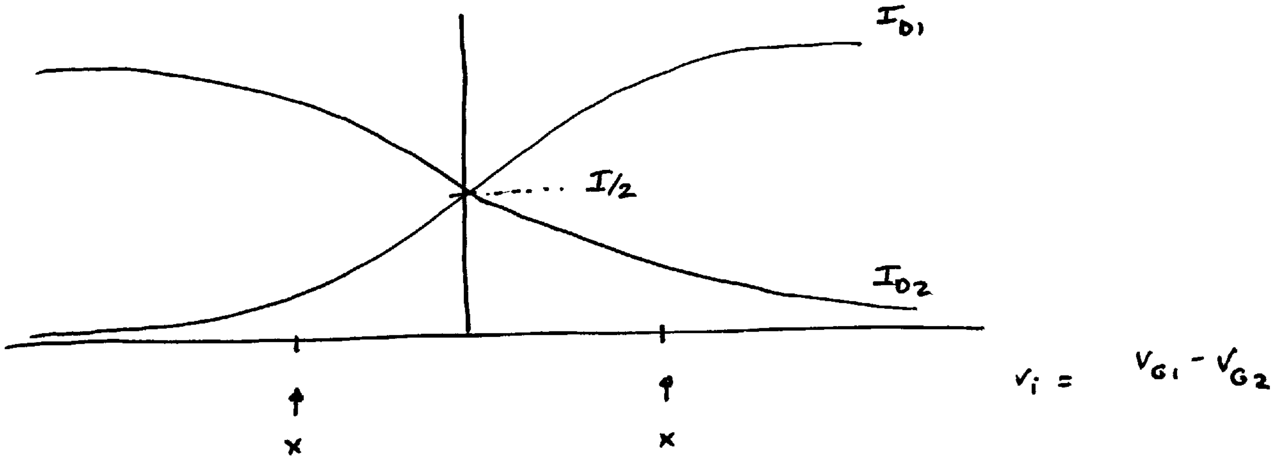
$$I_{D2} = \frac{I}{2} - \sqrt{KI}\, \frac{v_i}{2} \sqrt{1 - \frac{(v_i/2)^2}{I/K}}$$

note : $\frac{I}{2} = \frac{K}{2} (V_{GS\emptyset} - V_T)^2$ (when $V_{GS1} = V_{GS2}$; bias)

$$I_{D1} = \frac{I}{2} + \frac{I}{2}\left(\frac{v_i}{V_{GS0} - V_T}\right)\sqrt{1 - \frac{1}{2}\left(\frac{v_i}{V_{GS\phi} - V_T}\right)^2}$$

$$I_{D2} = \frac{I}{2} - \frac{I}{2}\left(\frac{v_i}{V_{GS\phi} - V_T}\right)\sqrt{1 - \frac{1}{2}\left(\frac{v_i}{V_{GS\phi} - V_T}\right)^2}$$

offset   linear   non-linear



x : when $\frac{1}{2}\left(\frac{v_i}{V_{GS\phi} - V_T}\right)^2$ becomes significant relative to 1

$$\frac{1}{2}\left(\frac{v_i}{V_{GS0} - V_T}\right)^2 \sim 1 \Rightarrow v_i \approx \sqrt{2} \times (V_{GS\phi} - V_T)$$

This is easily in the volts range. compare this to the bipolar transistor that operates in the 10's of mV range.