Figure 69 shows the circuit of the feedback loop β for the Colpitts Oscillator. The resistance R is the output resistance of the opamp. Ideally this is zero, but any real opamp has finite output resistance and this is essential, the feedback loop β is per definition the part of the output V_o that appears at the input V_i of our amplifier (that has a gain equal to $A = -R_\mathrm{f}/R_1$), that can be found by considering the voltage dividers in the circuit:

$$\beta \equiv \frac{V_\mathrm{i}}{V_\mathrm{o}} = \frac{Z}{Z+R} \times \frac{Z_\mathrm{C2}}{Z_\mathrm{C2}+Z_\mathrm{L}} \tag{206}$$

First, the second voltage divider composed of C_2 ($Z_\mathrm{C2} = 1/sC_2$) and L ($Z_\mathrm{L} = sL$), with $s = j\omega$ is given by

$$\frac{Z_\mathrm{C2}}{Z_\mathrm{C2}+Z_\mathrm{L}} = \frac{1}{1-\omega^2 LC_2}. \tag{207}$$

The impedance Z is given by

$$Z = \left[(1/sC_1)^{-1} + (1/sC_2 + sL)^{-1}\right]^{-1}. \tag{208}$$

Substituting this in (206) gives

$$\beta = \frac{1}{(1-\omega^2 LC_2) + j\omega RC_1[C_2/C_1 + (1-\omega^2 LC_2)]}. \tag{209}$$

Oscillation will occur when the loop gain $A\beta$ is unity. Since A is purely real, this means that β must be real too. This implies

$$RC_1[C_2/C_1 + (1-\omega^2 LC_2)] = 0. \tag{210}$$

Trivial solutions are $R = 0$ (ideal amplifier) or $C_1 = 0$. The non-trivial solution is

$$\omega = \sqrt{\frac{1}{L(C_1 \oplus C_2)}}, \tag{211}$$

with $C_1 \oplus C_2$ the series sum capacitance of C_1 and C_2, $C_1 \oplus C_2 = (1/C_1 + 1/C_2)^{-1}$. At this frequency the feedback loop is, according to Eq. (209), $\beta = -C_1/C_2$. Thus, if our amplifier has a gain A such that the total loop-gain is unity the Barkhausen criterion will be met; if

$$A\beta = \left(-\frac{R_\mathrm{f}}{R_1}\right) \times \left(-\frac{C_1}{C_2}\right) = 1 \tag{212}$$

the circuit will oscillate at the frequency given by

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{L(C_1 \oplus C_2)}}, \tag{213}$$

Note that the output resistance of the opamp does not enter into the final results. It only has effect in the *quality* of oscillation; the lower R the better ('sharper') the oscillation frequency.

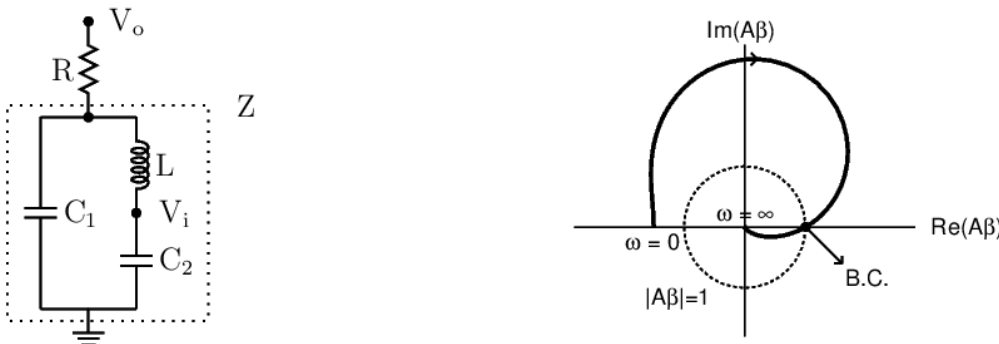

Fig. 69: Left: Circuit of the feedback loop of the Colpitts Oscillator. The resistance R is the output resistance of the opamp. Right: Nyquist plot for an example that meets the condition (Barkhausen Criterion, B.C.) of oscillation, for $R_f/R_1 = C_2/C_1$. (Exercise 16)

(excerpt from the upcoming book Electronic Instrumentation, P. Stallinga)