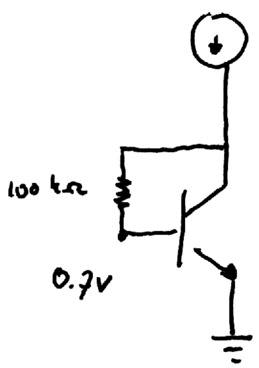
a)

$V_{out}$ $V_{in}$

$B\,\cancel{C}\,\cancel{E}$

$\hookrightarrow$ Common Base

b)




$$I_c + I_B = 2\ mA$$

$$I_E = I_B + I_c = 2mA$$

$$\beta = 99 \ , \quad I_c = \beta I_B$$

$$\beta I_B + I_B = 2\ mA$$

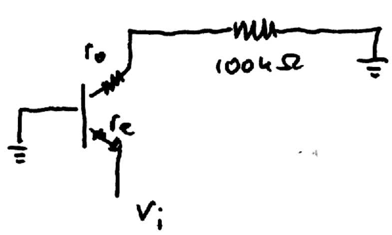
$$I_B = \frac{2\ mA}{\beta+1} = 20\ \mu A$$

$$I_c = 1.98\ mA \ , \quad I_E = 2.0\ mA$$

$V_E = 0$, $V_B = 0.7\ V$ $\quad$ $V_C = V_B + I_B \times 100\ k\Omega = 2.7\ V$

note : for DC analysis
- C = open circuit
- Signal sources to ground

c) For AC (small signal) analysis
- voltage sources : ground
- current sources : open circuit
- capacitors : short circuit $\quad$ —||— ⇒ ——



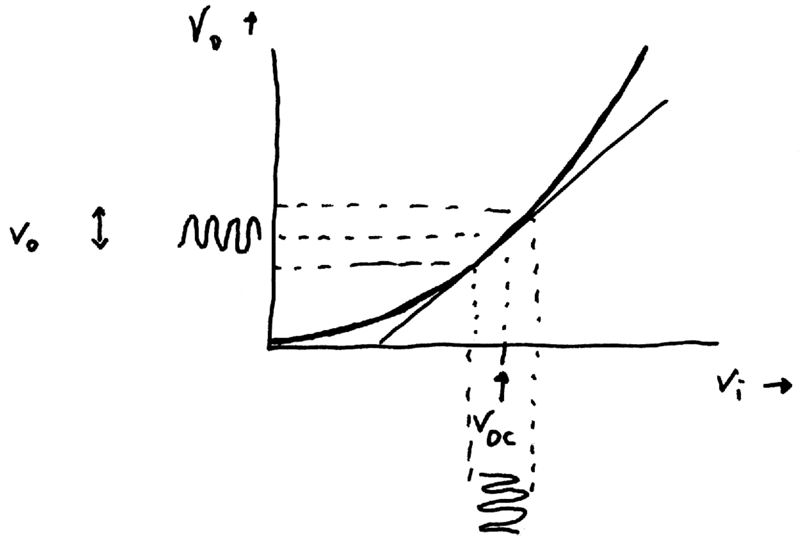

$$r_e = \frac{V_T}{I_E} = \frac{25\ mV}{1\ mA} = 25\ \Omega$$

$$r_o = \frac{V_A}{I_c} = \frac{100\ V}{2\ mA} = 50\ k\Omega$$

From Electronics I : Common Base Amp. Gain

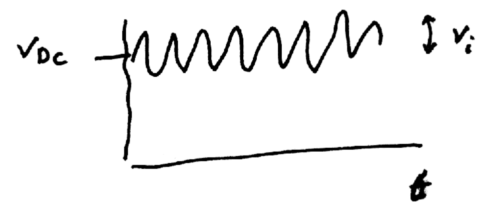
$$+\frac{\text{everything at collector}}{\text{everything at emitter}} = \frac{100\ k\Omega // 50\ k\Omega}{25\ \Omega} = 1333$$

Why gain is like this ?




$$V_i(t) = V_{DC} + v_i \sin(\omega t)$$

$$V_o(t) = V_{DC} + v_o \sin(\omega t + \phi)$$




$$v_o = \frac{dV_o}{dV_i} \times v_i$$

small signal

$$A_v \equiv \frac{v_o}{v_i} = \frac{dV_o(V_i)}{dV_i}$$

$$= \frac{dV_o}{dI_c} \cdot \frac{dI_c}{dI_B} \cdot \frac{dI_B}{dV_i} \qquad (*)$$

$\frac{dI_B}{dV_i}$ :

Ebers Moll

$$I_c = I_o \left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] + \cdots$$

$$\simeq I_o \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$I_B = \frac{I_c}{\beta} = \frac{I_o}{\beta} \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$= \frac{I_o}{\beta} \exp\left(-\frac{V_i}{V_T}\right)$$

$V_{BE} = V_B - V_E = -V_i$

B = ground

E = input

$$\frac{dI_B}{dV_i} = \frac{I_o}{\beta} \exp\left(-\frac{V_i}{V_T}\right) \times \left(-\frac{1}{V_T}\right) = -\frac{I_B}{V_T} = -\frac{I_C}{\beta V_T} = -\frac{I_E}{(\beta+1) V_T}$$

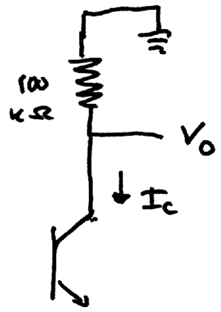
$$\equiv -\frac{1}{(\beta+1)} \cdot \frac{1}{r_e} = -\frac{1}{r_\pi}$$

$r_e$ and $r_\pi$ are parameters (virtual resistances) to describe the behavior

$$\frac{dI_B}{dV_i} = -\frac{1}{r_\pi} = -\frac{1}{\beta+1} \cdot \frac{1}{r_e} = -\frac{g_m}{\beta+1}$$

$$\frac{dI_c}{dI_B} = \beta$$

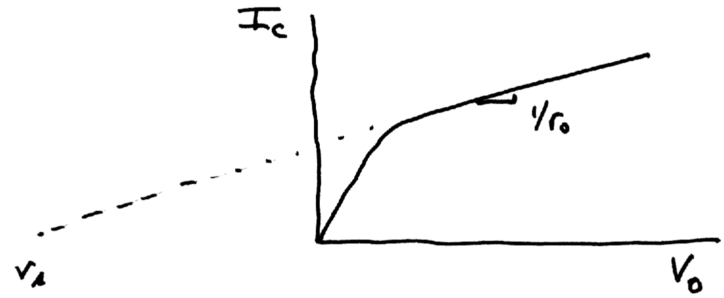
$\frac{dV_o}{dI_c}$ :




Forgetting $r_o$ of transistor, it is obvious that

$$\frac{dV_o}{dI_c} = -100\ k\Omega$$

Including $r_o$ of transistor




$$\frac{dI_c}{dV_o} \equiv \frac{1}{r_o} = \frac{1}{50\,k\Omega}$$

$$\Rightarrow \frac{dV_o}{dI_c} = 50\ k\Omega$$

combining two effects : $\frac{dV_o}{dI_c} = 50\ k\Omega // 100\ k\Omega$

$= 33.3\ k\Omega$

Everything in (*) :

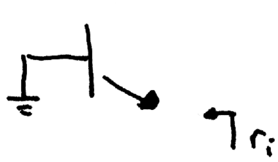
$$A_v = \frac{dV_o}{dI_c} \cdot \frac{dI_c}{dI_B} \cdot \frac{dI_B}{dv_i} = (-33.3\ k\Omega) \times (\beta) \times \left(-\frac{g_m}{\beta+1}\right)$$

everything at collector

$$= \frac{33.3\ k\Omega}{r_e} \times \frac{\beta}{\beta+1}$$

everything at emitter

d) $r_i \equiv 1 / \frac{dI_i}{dv_i}$



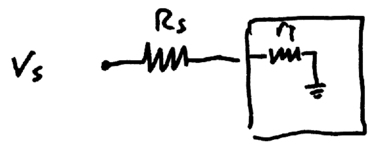

$$I_i = -I_E = -I_{EO} \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$= -I_{EO} \exp\left(-\frac{V_i}{V_T}\right)$$

$$\frac{dI_i}{dV_i} = -I_{EO} \exp\left(-\frac{V_i}{V_T}\right) \cdot \left(-\frac{1}{V_T}\right) = \frac{I_{EO}}{V_T} = \frac{1}{r_e} = g_m$$

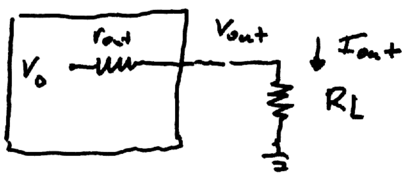
$r_i = r_e$

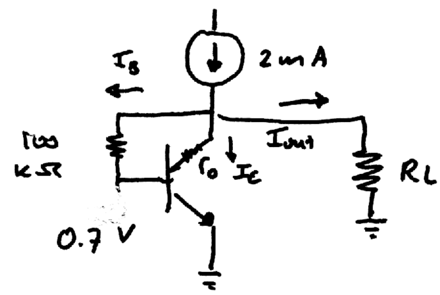
$r_i$ represents the losses due to the non-infinite input resistance (ideal: $r_{in} = \infty$, $i_{in} = 0$, $v_{in}$ unaltered)




Thevenin Equivalent

e) $r_{out}$ represents if loading the amplifier ($I_{out} \neq 0$) changes the signal $V_{out}$ or not.




If $R_L = \infty$ : $V_{out} = V_o = 2.7\ V$

$I_{out} = \emptyset$

$V_{out}$ is a function of $I_{out}$ $V_{out}(I_{out})$

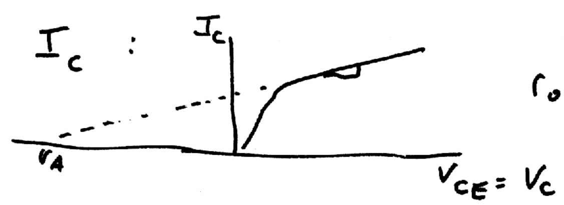
$$r_{out} = \frac{d V_{out}}{d I_{out}} = 1 \Big/ \frac{d I_{out}}{d V_{out}}$$

$$I_{out} = 2\ mA - I_B - I_c$$

$$\frac{d I_{out}}{d V_{out}} = \frac{d I_{out}}{d V_c} = 0 - \frac{1}{100\ k\Omega} - \frac{1}{r_o}$$

$$I_B = \frac{V_c - V_B}{100 k\Omega} = \frac{V_c - 0.7 V}{100 k\Omega}$$

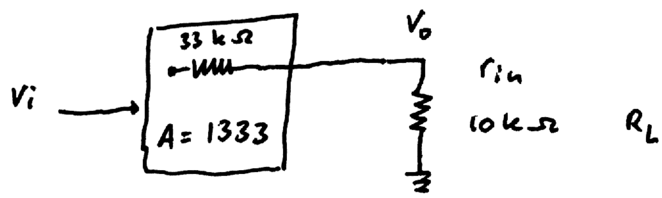
$I_c$ :




$$r_o = \frac{V_A}{I_c} = 50\ k\Omega$$

$$r_{out} = 1 \Big/ \frac{d I_{out}}{d V_{out}} = \frac{1}{1/100 k\Omega + 1/r_o} = 100\ k\Omega \,//\, 50\ k\Omega = 33.3\ k\Omega$$

$r_{out}$ is important because we will be able to calculate the gain when we connect a load with finite impedance




<u>without</u> knowing anything (else) about our amplifier, with the parameter $r_{out}$ (= 33 kΩ) we can calculate the gain of the total circuit

$$\frac{V_o}{V_i} = A_V \times \frac{R_L}{R_L + r_{out}} = 1333 \cdot \frac{10\ k\Omega}{10\ k\Omega + 33\ k\Omega} = 310$$

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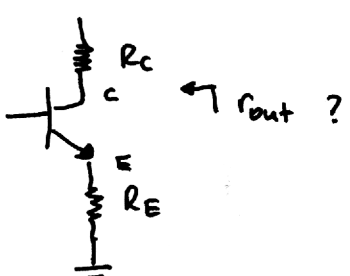
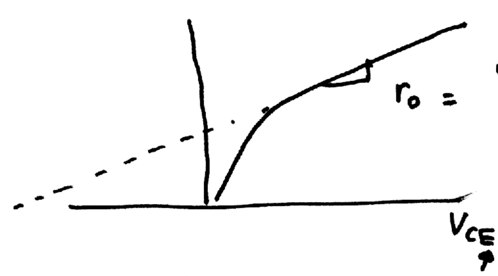
NOTE

Careful with calculation of (output) resistance when feedback takes place. Imagine the circuit below:




$r_o = 1 / \frac{dI_C}{dV_{CE}} = 100k\Omega$

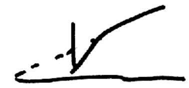
E is not ground!

what is total output resistance? One might think

$r_{out} = (r_o + R_E) \,//\, R_C$

WRONG! Why? Feedback!

Feedback . we want to know $\frac{dI_c}{dV_c}$

- Imagine $V_c$ raised
- $I_c$ rises ( Early voltage → $r_o$ , etc. ) 
- $I_E$ rises
- $V_E$ rises ( $V_E = I_E \times R_E$ )
- $V_B - V_E$ drops !!!
- $I_B$ drops ( Ebers - Moll $I_o\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right]$ )
- $I_E = (\beta+1) \cdot I_B$ and $I_c = \beta I_B$ drop !!

$\Downarrow$

$I_c$ rises <u>less</u> than expected

$\frac{dI_c}{dV_c}$ is less than expected

$r_{out}$ is much larger than expected.

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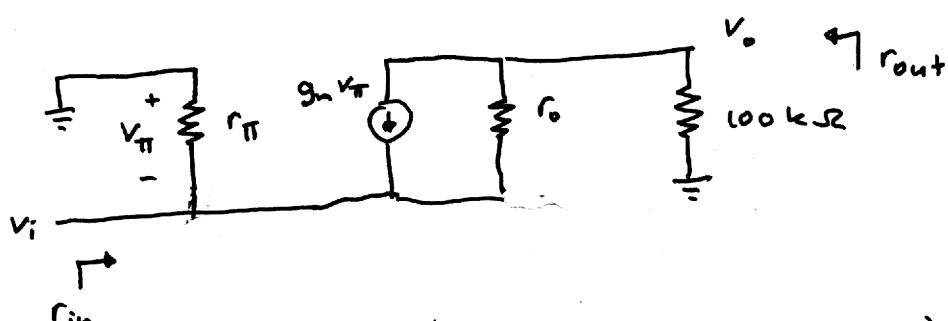
small signal equivalent ($\pi$ - hybrid)




$r_{in} \neq r_\pi \,/\!\!/\, (r_o + 100\,k\Omega)$ !

$r_{out} = 100\,k\Omega \,/\!\!/\, r_o$