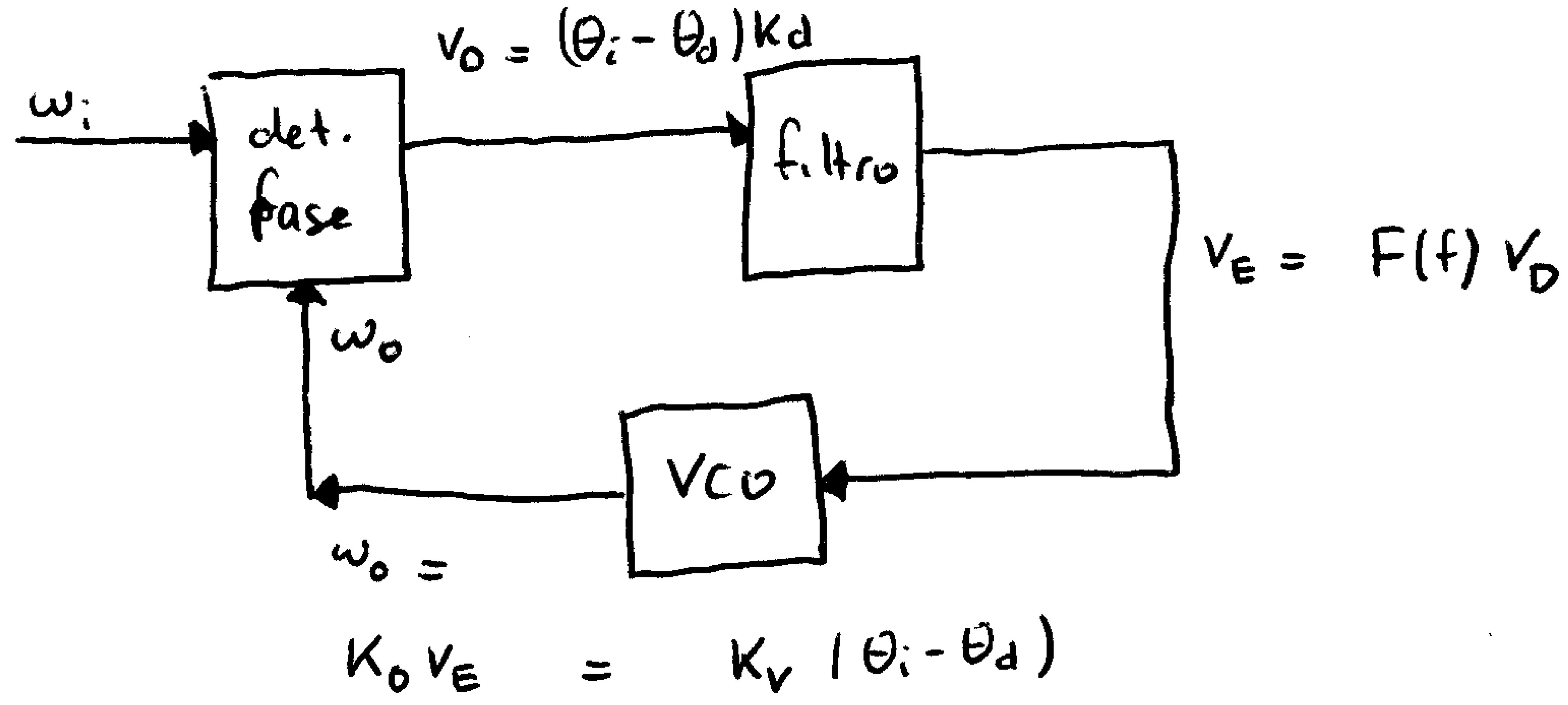
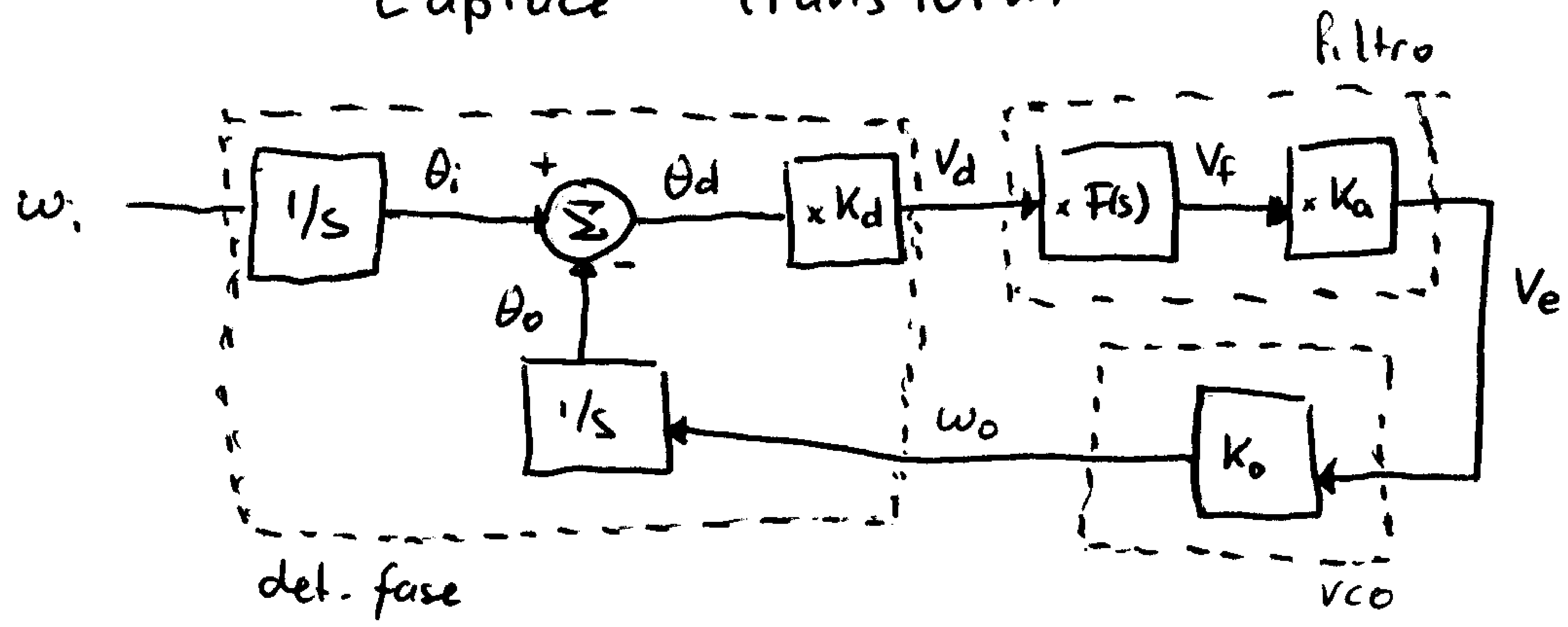



Laplace transform:




nota1: integration in time domain is equal to division by s in the frequency domain

nota 2: $\theta(t) = \int_0^t \omega(\tau)\, d\tau \Rightarrow \theta_i = \omega_i / s$

$$\theta_d = \theta_i - \theta_o$$

$$V_d = K_d\, \theta_d = K_d (\theta_i - \theta_o)$$

$$V_f = F(s) \cdot V_d = F(s)\, K_d\, (\theta_i - \theta_o)$$

$$V_e = K_a V_f = K_a K_d\, F(s) (\theta_i - \theta_o)$$

$$\omega_o = K_o V_f = K_o K_a K_d\, F(s)\, (\theta_i - \theta_o)$$

$$\theta_o = \omega_o / s = K_o K_a K_d\, F(s)\, (\theta_i - \theta_o) / s = K_v \frac{F(s)}{s} (\theta_i - \theta_o)$$

em malha aberta esto $\theta_o = 0 \Rightarrow$

$$T(s) = \frac{\theta_o}{\theta_i} = \frac{K_v F(s)}{s} \quad \text{(malha aberta)}$$

Malha fechada :

$$H(s) = \frac{\theta_o}{\theta_i} = \frac{K_v F(s)}{s + K_v F(s)}$$

nota 3 :

$K_v = K_o K_a K_d$

sem filtro : $K_a = 1$ e

$K_o = K_v / K_d$ , $K_d = \frac{K_v}{K_o}$

mais importante :

$$\boxed{\frac{V_e}{\omega_i} = \frac{1}{K_o} \cdot \frac{K_v F(s)}{s + K_v F(s)}}$$

porque $\theta_i = \omega_i(s)/s$

$\theta_o = \frac{K_o}{s} V_e(s)$

---

Exemplo : $K_v = 500\ s^{-1}$

MALHA ABERTA

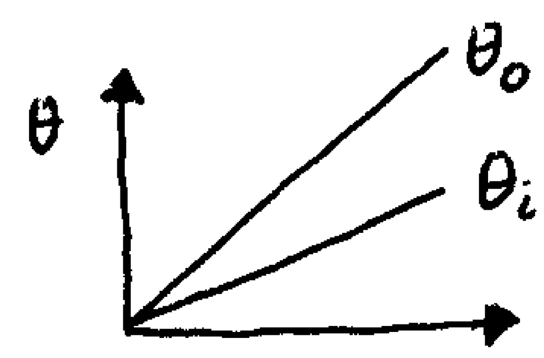
$F(s) = 1$

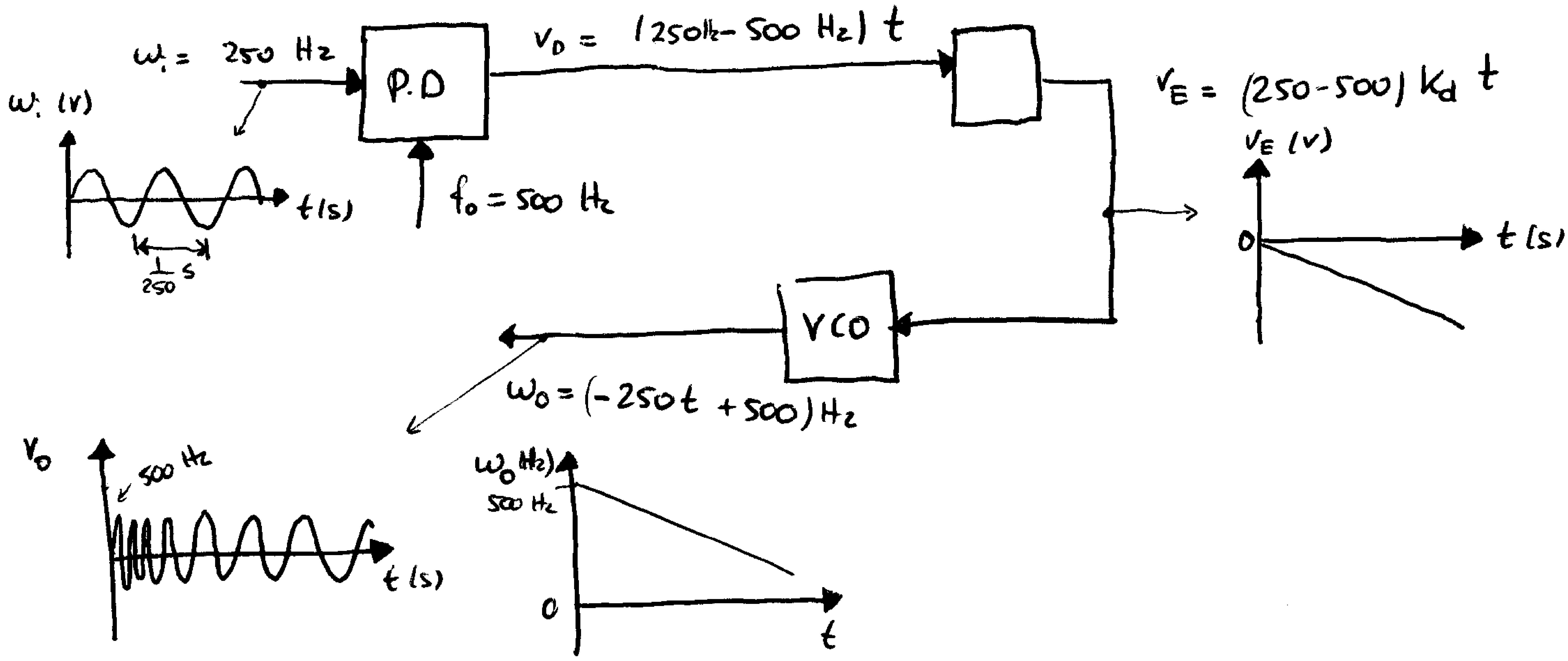
$f_o = 500$ Hz

$K_o = 1$ kHz /V

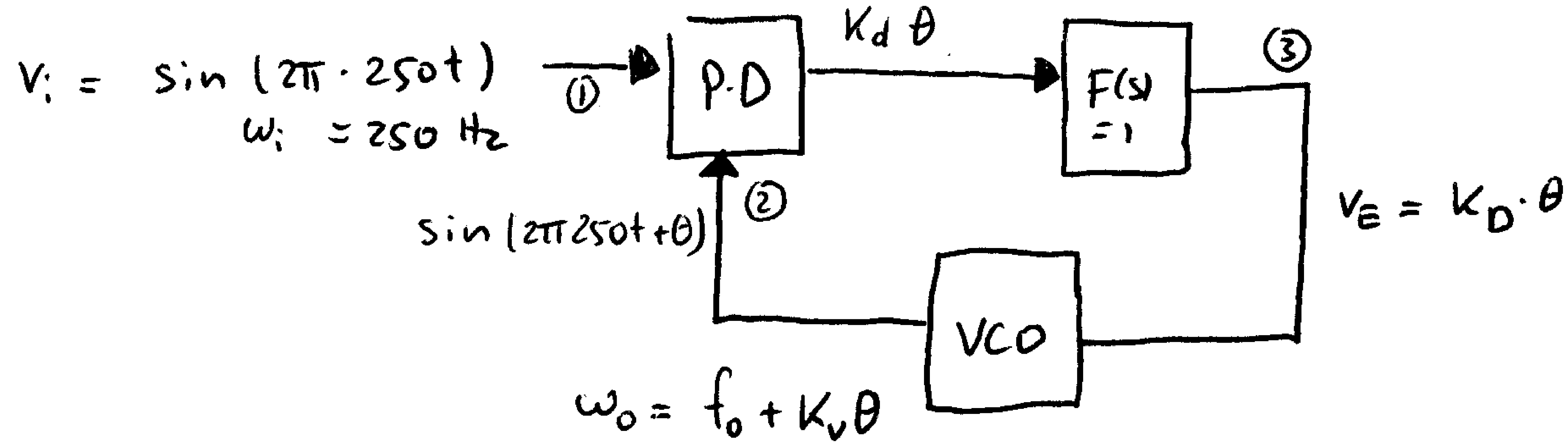
$\Rightarrow K_d = \frac{K_v}{K_o} = \frac{500 \text{ Hz}}{1 \text{ kHz/V}} = 0.5$ V

$\omega_i = 250$ Hz



MALHA FECHADA:




temos "lock" frequências de entrada (250 Hz) e saído são iguais.

$\boxed{\text{só há uma diferencia de fase : } \theta}$

vamos calcular $\theta$:

$$\omega_i = \omega_o$$

$$\omega_i = f_o + K_v\,\theta \implies \theta = \left(\frac{\omega_i - f_o}{K_v}\right)$$

$$V_E = K_d\theta = K_d\left(\frac{\omega_i - f_o}{K_v}\right)$$

$f_o = 500$ Hz, $K_v = 500$ Hz, $K_o = 1$ kHz/V

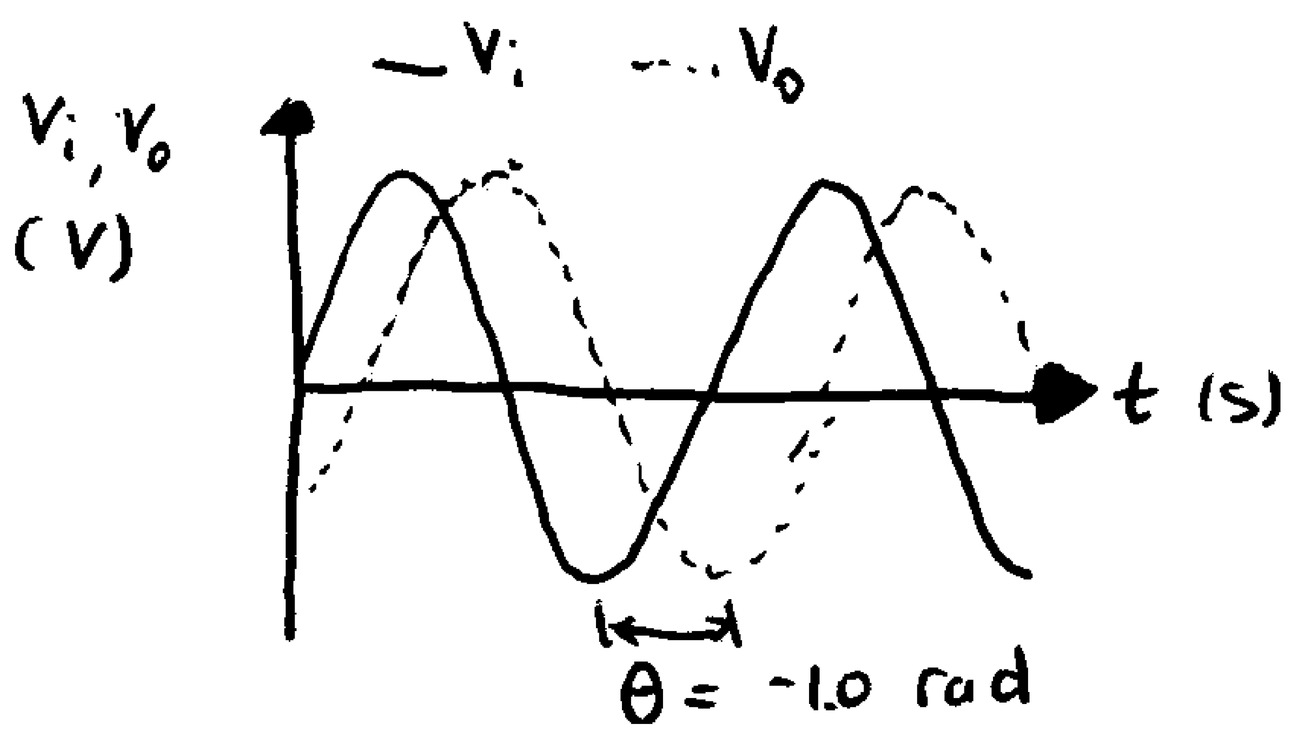
$K_d = K_v/K_o = (500\text{ Hz})/(1\text{ kHz/V}) = 0.5\text{ V}$

$$\omega_i = 250\text{ Hz} \implies \theta = \frac{(250-500)\text{Hz}}{500\text{ Hz}} = -0.5\text{ rad}$$

$$V_E = K_d\left(\frac{\omega_i - f_o}{K_v}\right) = 0.5 \cdot \frac{250\text{ Hz} - 500\text{ Hz}}{500\text{ Hz}} = -0.25\text{ V}$$

$$\omega_i = 1\text{ kHz} \implies \theta = \frac{1\text{ kHz} - 500\text{ Hz}}{500\text{ Hz}} = -1.0\text{ rad}$$

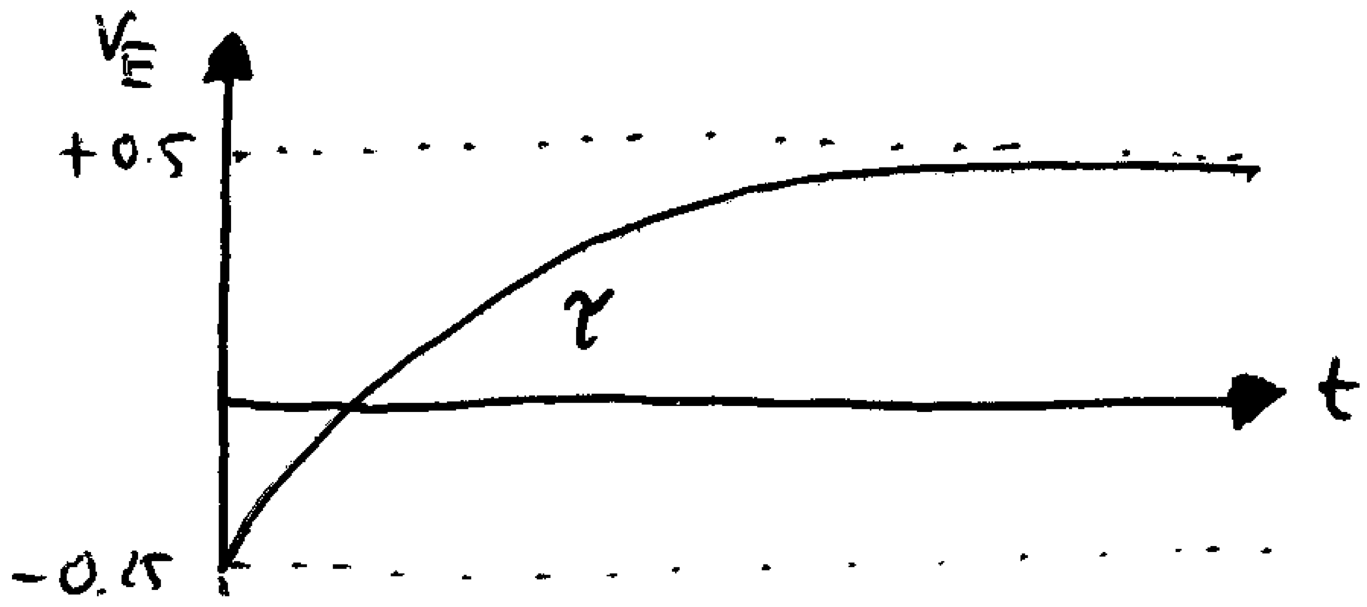
$$V_E = 0.5\,\frac{1\text{kHz} - 500\text{ Hz}}{500\text{ Hz}} = +0.50\text{ V}$$



1c) $\omega_i = 250\ Hz \rightarrow V_E = -0.25V$

$\omega_i = 500\ Hz \rightarrow V_E = +0.5V$

quando $\omega_i$ muda instantaneamente entre 250 Hz e 1kHz temos a resposta $V_E$ :




Em malha fechada :

$$\frac{V_e}{\omega_i} = \frac{1}{K_o} \frac{K_v F(s)}{s + K_v F(s)}$$ . sem Filtro $F(s)=1$ :

$$\frac{V_e}{\omega_i} = \frac{1}{K_o} \frac{K_v}{s + K_v} = \frac{1}{K_o} \cdot \frac{1}{1 + s/K_v}$$

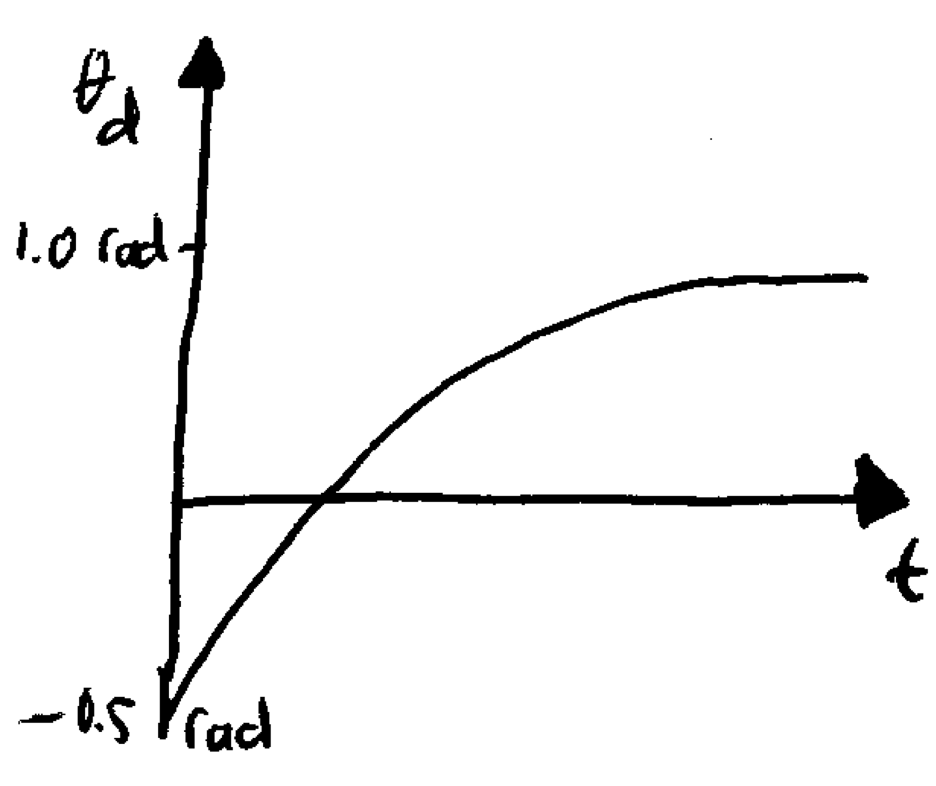
isto é um filtro pasa baixa! $f_c = \frac{K_v}{2\pi}$

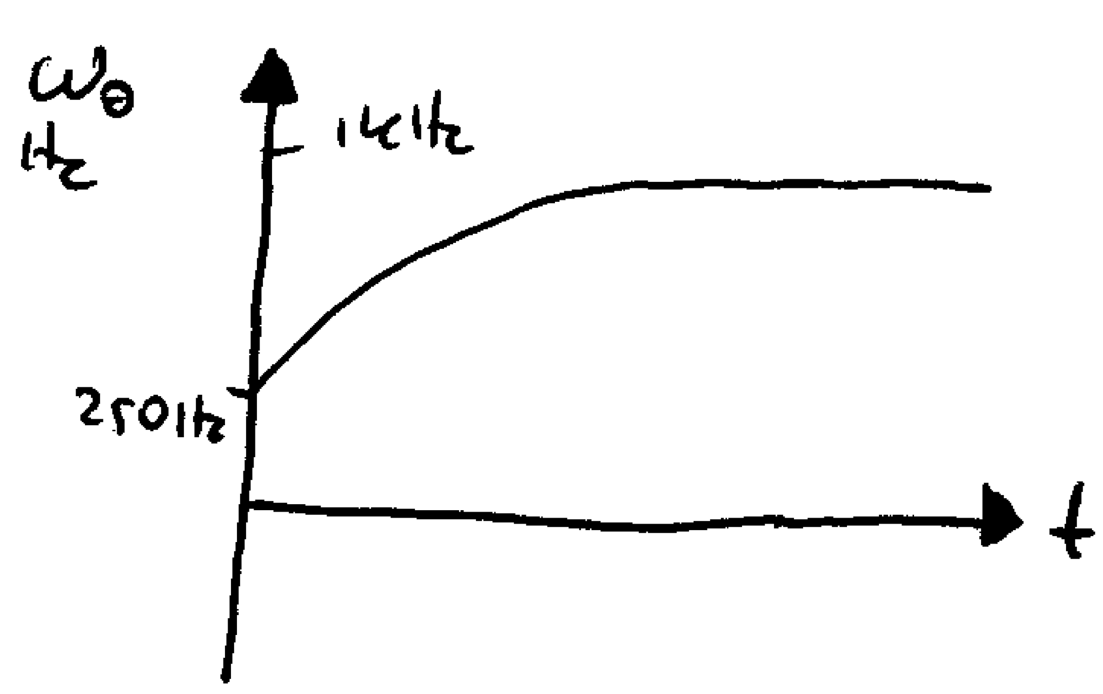
teorema

| filtro pasa baixa | $\Longleftrightarrow$ | resposta exponente aos sinais ⌐_ |
|---|---|---|
| $f_c$ = frequencia de corte | | $\tau = \frac{1}{2\pi f_c}$ |

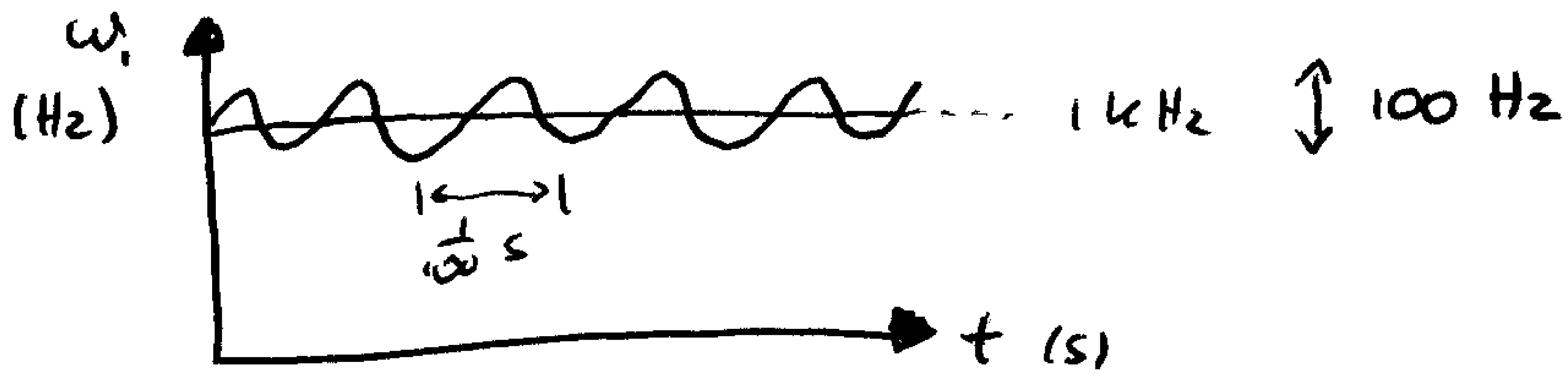
neste caso :

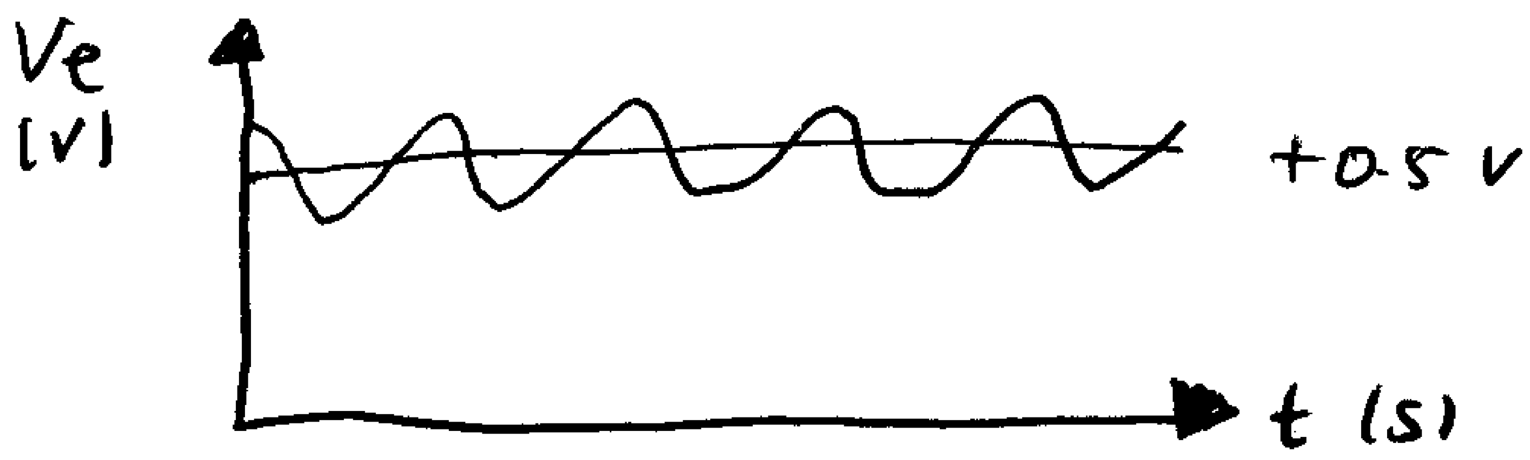
$$f_c = \frac{K_v}{2\pi} \Rightarrow \tau = \frac{1}{2\pi f_c} = \frac{1}{K_v} = \frac{1}{500\ Hz} = 2\ ms$$



1d $\omega_i = 1000\ (1 + 0.1 \sin(200\pi t))$




$$\omega_i = \overbrace{(1\ kHz)}^{DC} + \overbrace{(100\ Hz)\cdot \sin(100 \cdot 2\pi t)}^{Ac}$$

$$\frac{V_e}{\omega_i} = \frac{1/K_o}{1 + j\omega/K_V}$$

DC : $\omega = 0 \quad \frac{V_e}{\omega_i} = \frac{1}{K_o} \Rightarrow V_e = 0.5\ V$

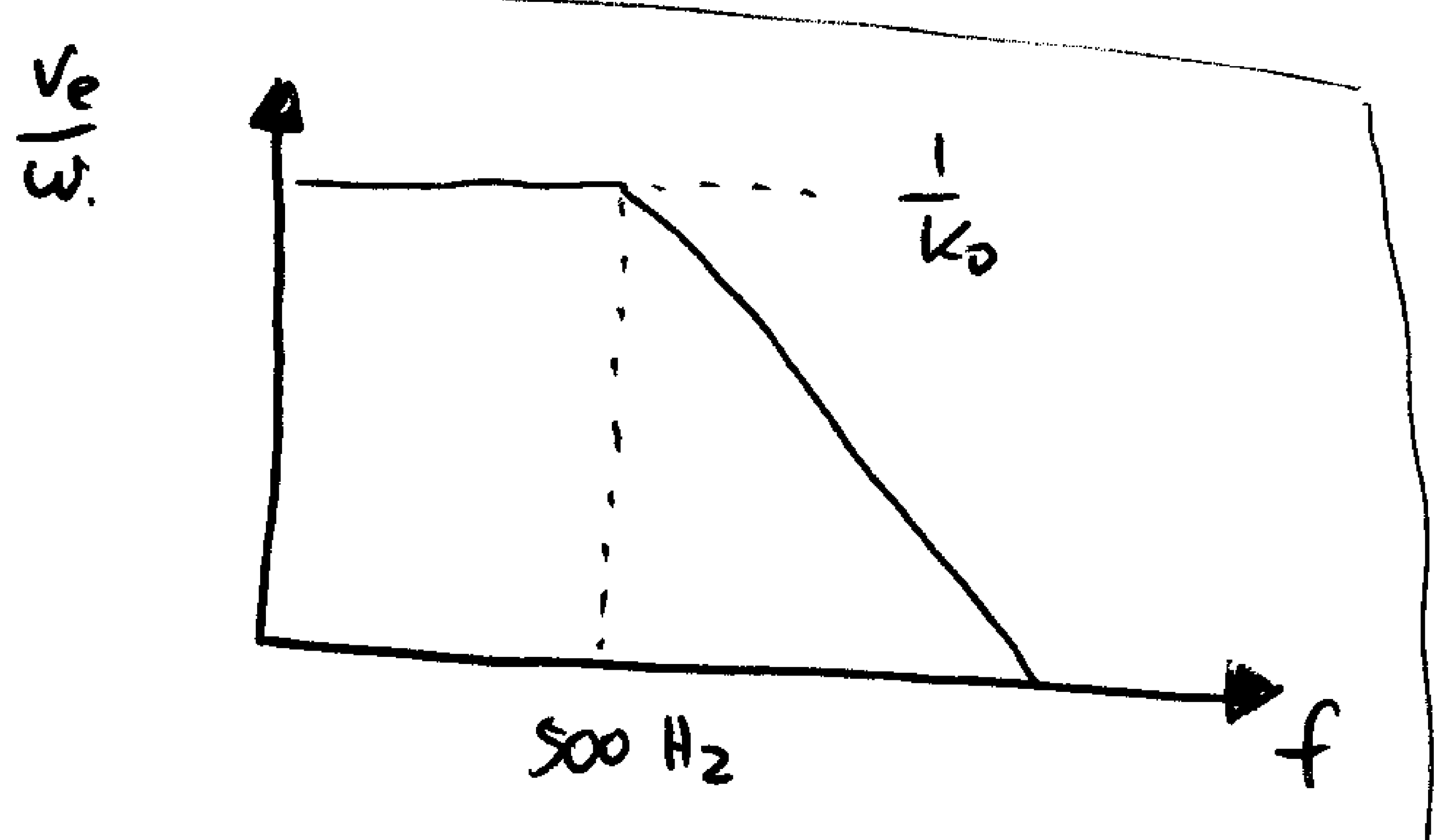
Ac : $\frac{V_e}{\omega_i} = \frac{1}{K_o} \cdot \frac{K_v}{K_v + jf}$ $\quad f = 100\ Hz$, $\omega_i = 100\ Hz$

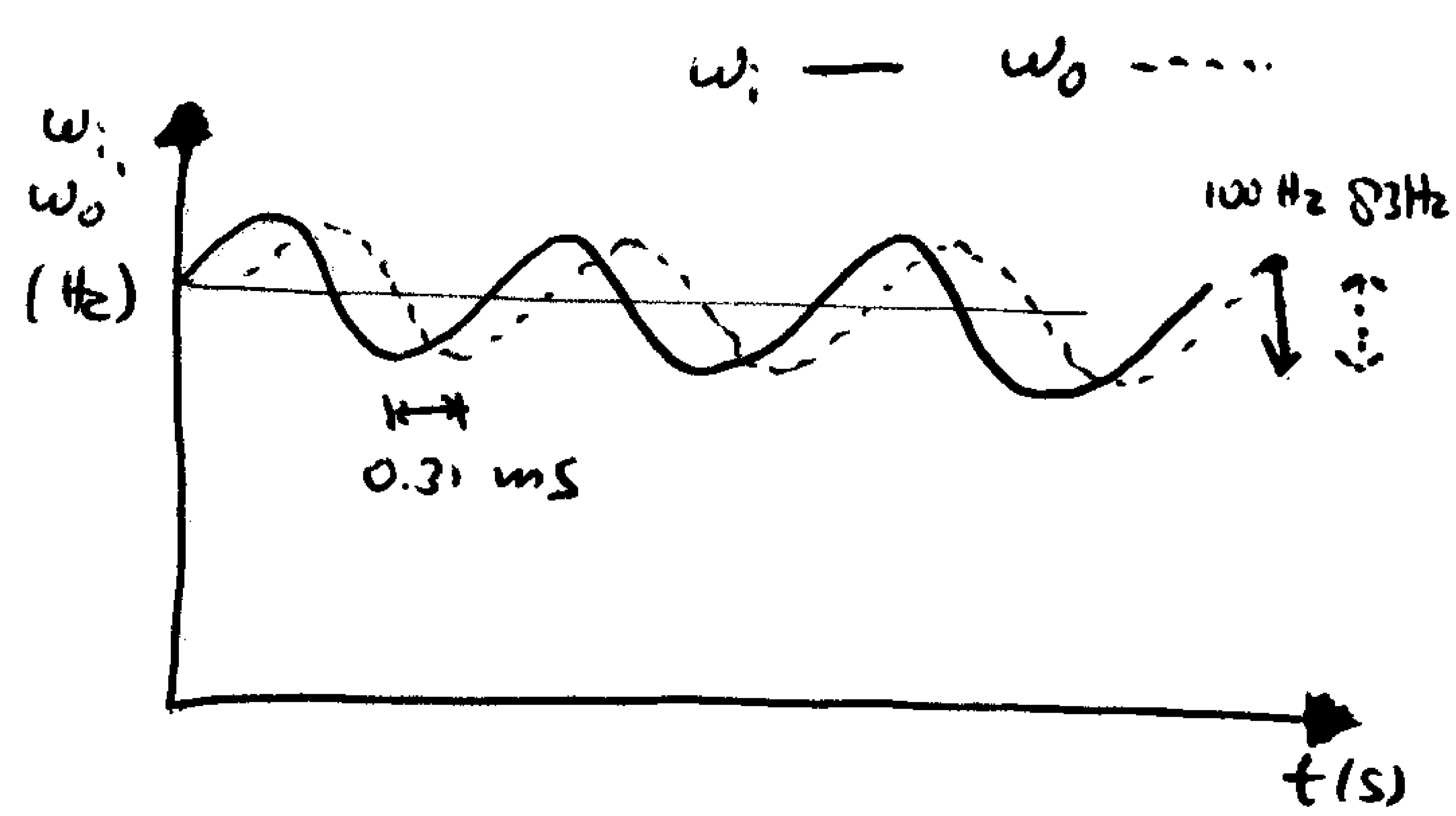
$$\left\{\begin{array}{l} |V_e| = \frac{1}{1\ kHz/V} \cdot \frac{500\ Hz}{500\ Hz + 100\ Hz} \cdot 100\ Hz = 83\ mV \\ \phi = \tan^{-1}(f/K_v) = 11.3^\circ \end{array}\right.$$

$$V_e = (0.5\ V) + (83\ mV)\cdot \sin(100 \cdot 2\pi t - 11.3^\circ)$$

$$= (0.5\ V) + (83\ mV)\cdot \sin(100 \cdot 2\pi (t - \Delta t))$$

$\Delta t = 0.31\ ms$



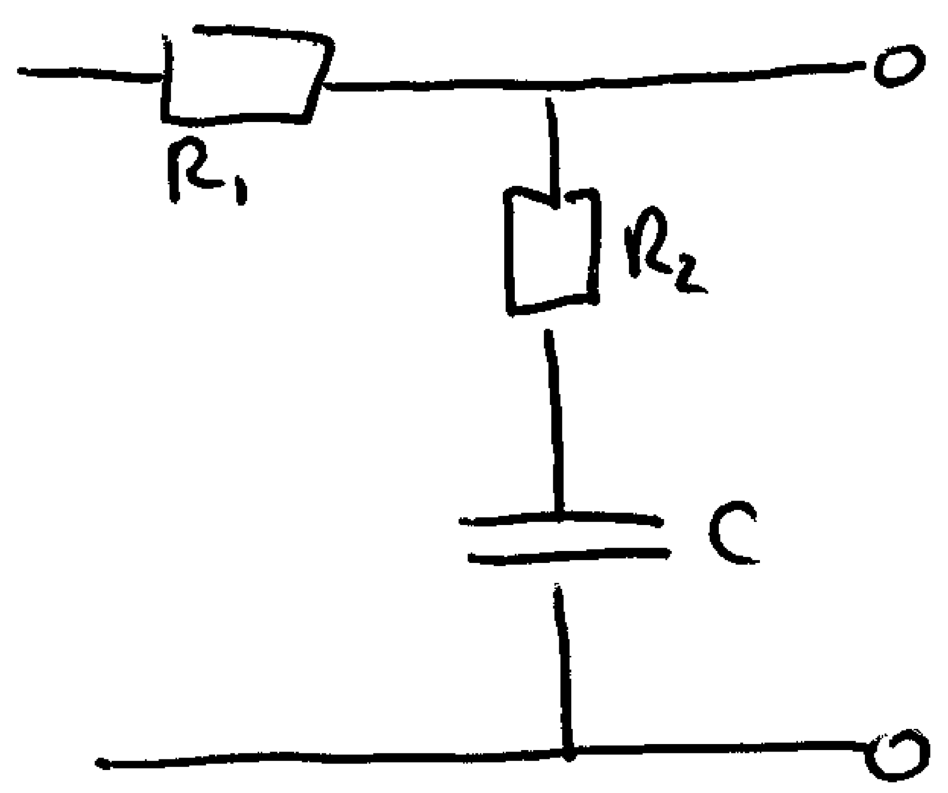



$$\omega_0 = K_0 V_E = \frac{K_v}{f+K_v} \cdot \omega_i =$$

$$\frac{500\ Hz}{100\ Hz + 500\ Hz} \times 100\ Hz = 83\ Hz$$

(amplitudo de sinal de saida)

2)

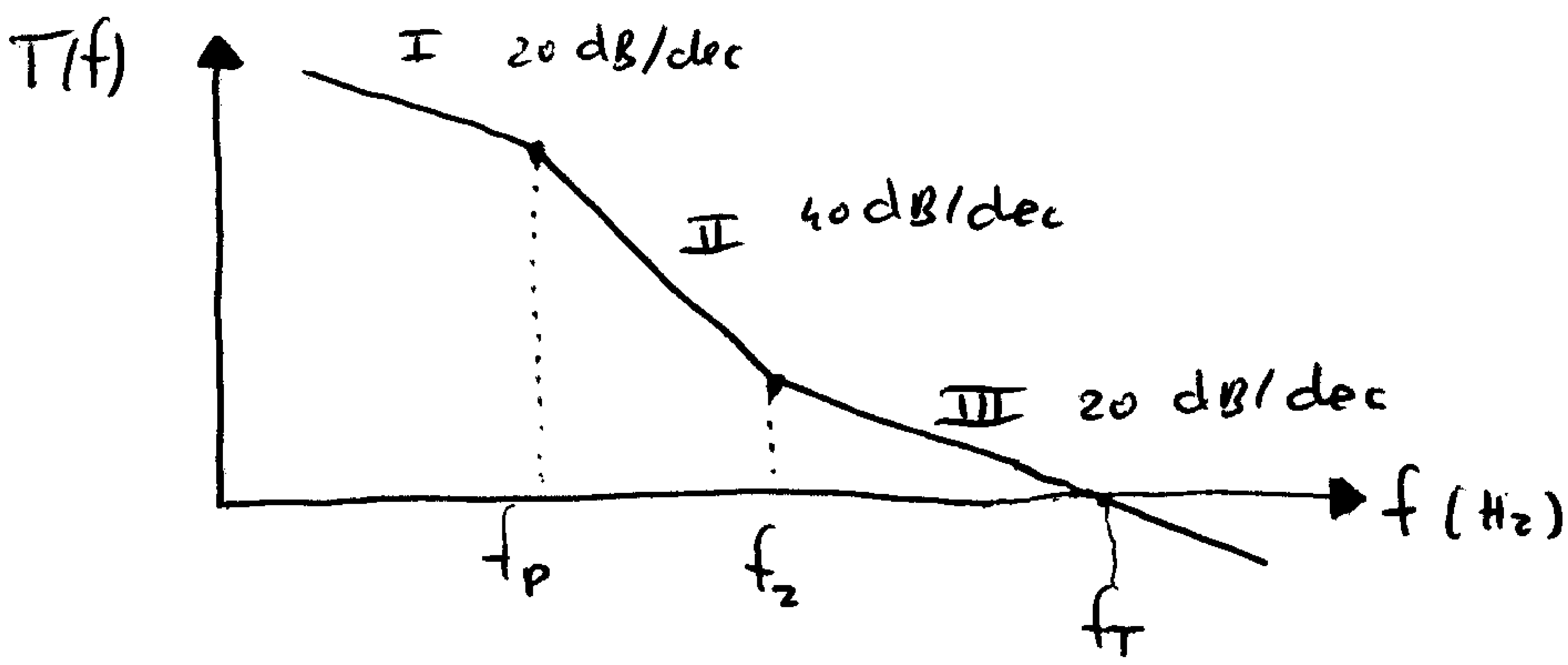



filtro "lag-lead"

$$F(s) = \frac{1 + s/f_z}{1 + s/f_p}$$

$$f_z = \frac{1}{2\pi R_2 C}, \quad f_p = \frac{1}{2\pi (R_1+R_2)C}$$

$$T(s) = K_v \frac{F(s)}{s}$$ → polo $f_p$, zero a $f_z$ → polo a 0 Hz




$$\mathrm{I}: |T(f)| = \frac{K_v}{f}$$

$$\mathrm{II}: |T(f)| = \frac{K_v}{f} \cdot \frac{1}{1+f/f_p} \sim \frac{K_v}{f^2} f_p$$

$$\mathrm{III}: |T(f)| = \frac{K_v}{f} \cdot \frac{1+f/f_z}{1+f/f_p} \sim \frac{K_v}{f} \cdot \frac{f_p}{f_z}$$

Queremos uma margem de fase 45° $\Rightarrow$

$$f_z = f_T \qquad ①$$

queremos $f_T = 1000 \text{ Hz}$ ②

①: $f_z = f_T$ : $\frac{1}{2\pi R_2 C} = f_T = 10^3 \Rightarrow R_2 C = \frac{1}{2000\pi}$

②: $T(f_T) = 1 \overset{\text{II}}{\Rightarrow} \frac{K_v}{f_T^2} \cdot f_p = 1 \Rightarrow f_p = \frac{f_T^2}{K_v}$

$$f_p = \frac{(10^3)^2}{10^4} = 100 \text{ Hz}$$

$$f_p = \frac{1}{2\pi (R_1 + R_2) C} = 100 \text{ Hz}$$

②: $(R_1 + R_2) C = \frac{1}{200\pi}$

①: $R_c C = \frac{1}{2000\pi}$

$$C = 1 \text{ nF} \overset{①}{\Rightarrow} R_2 = 159.1 \text{ k}\Omega$$

$$\overset{②}{\Rightarrow} R_1 + R_2 = 1.591 \text{ M}\Omega$$

$$\Rightarrow R_1 = 1.432 \text{ M}\Omega$$