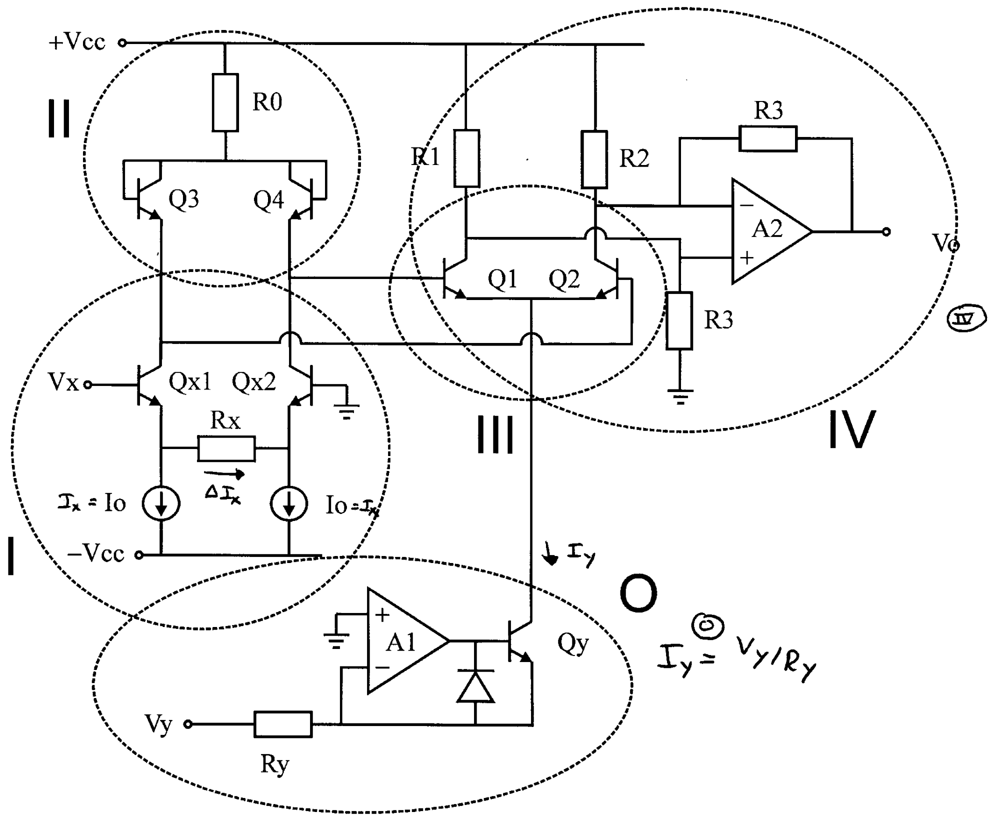
F.P. 5 E - III

Ⅲ $\frac{I_c + \Delta I_c}{I_c - \Delta I_c} = \frac{I_x + \Delta I_x}{I_x - \Delta I_x} \Rightarrow \frac{\Delta I_c}{I_c} = \frac{\Delta I_x}{I_x}$

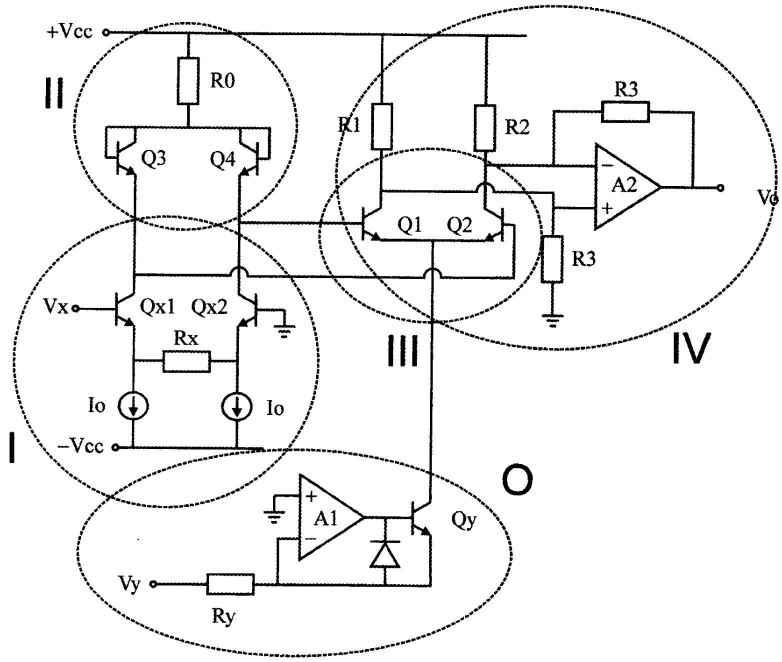
Ⅱ $V_{be_3} - V_{be_4} = V_T \log \frac{I_x + \Delta I_x}{I_x - \Delta I_x}$

Ⅰ $\Delta I_x = \frac{V_x}{R_x}$



Ⅳ $V_o = -2 R_3 \Delta I_c$

⓪ $I_y = V_y / R_y$

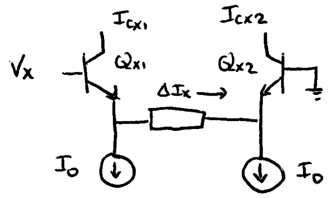


Igual a p. 32 de sebenta a) : 2 quadrantes (veja p. 6)

(O) Fonte de corrente : $I_y = \alpha \frac{V_y}{R_y} \approx \frac{V_y}{R_y}$

(I) p. 37 de sebenta

«e»



$V_x = 0 \Rightarrow$ simetria

$$V_{bx_1} = V_{bx_2}$$

$$V_{ex_1} = V_{ex_2}$$

$$\Downarrow$$

$$\Delta I_x = 0$$

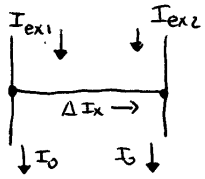
$V_x \neq 0$, assumindo $V_{ebx_1} = V_{ebx_2} = 0.7\ V$

$$\Rightarrow V_{ex_1} = V_{ex_2} + V_x$$

$$\Rightarrow \Delta I_x = \frac{V_x}{R_x}$$

F.P. 5 E-III

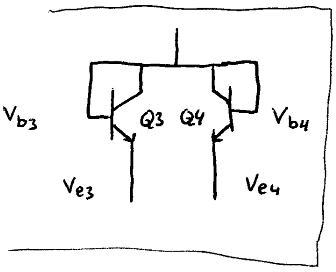
Kirchov



$$I_{ex1} = I_0 + \Delta I_x$$

$$I_{ex2} = I_0 - \Delta I_x$$

II Q3 e Q4 são diodos



$$I_{e3} = I_s\left[\exp\left(\frac{V_{be3}}{V_T}\right) - 1\right] = I_x + \Delta I_x$$

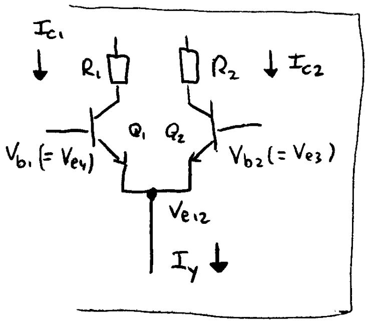
$$I_{e4} = I_s\left[\exp\left(\frac{V_{be4}}{V_T}\right) - 1\right] = I_x - \Delta I_x$$

negligible

$$\frac{I_x + \Delta I_x}{I_x - \Delta I_x} = \exp\left(\frac{V_{be3} - V_{be4}}{V_T}\right)$$

$$V_{be3} - V_{be4} = V_T \log\left(\frac{I_x + \Delta I_x}{I_x - \Delta I_x}\right)$$

III $V_{e1} - V_{e2} = 0$ $\quad I_c = I_Y/2$ $\quad$ «d»



$$I_{c1} = I_s \exp\left(\frac{V_{b1} - V_{e1}}{V_T}\right) \equiv I_c + \Delta I_c$$

$$I_{c2} = I_s \exp\left(\frac{V_{b2} - V_{e2}}{V_T}\right) \equiv I_c - \Delta I_c$$

$$V_{e1} = V_{e2} \quad (\equiv V_{e12})$$

$$\Rightarrow \quad V_{be1} - V_{be2} = V_{b1} - V_{b2} = V_{e4} - V_{e3}$$

$$= V_T \log\left(\frac{I_x + \Delta I_x}{I_x - \Delta I_x}\right)$$

$$\frac{I_c + \Delta I_c}{I_c - \Delta I_c} = \frac{I_{c1}}{I_{c2}} = \exp\left(\frac{V_T}{V_T} \log\left(\frac{I_x + \Delta I_x}{I_x - \Delta I_x}\right)\right) = \frac{I_x + \Delta I_x}{I_x - \Delta I_x}$$

Intermezzo : Taylor $f(x)$, $x \approx 0$

$$f(x) = \frac{1}{0!} f(0) + \frac{1}{1!} f'(0) \cdot x + \frac{1}{2!} f''(0)\, x^2 + \frac{1}{3!} f'''(0)\, x^3 \ldots$$

$$f(x) = \frac{1+x}{1-x} \quad : \quad f(0) = 1$$

$$f'(x) = \frac{2}{(1-x)^2} \quad : \quad f'(0) = 2$$

$$\Rightarrow \quad \frac{1+x}{1-x} = 1 + 2x + O(x^2)$$

$$\frac{I_c + \Delta I_c}{I_c - \Delta I_c} = \frac{1 + (\Delta I_c / I_c)}{1 - (\Delta I_c / I_c)} = 1 + 2 \cdot \frac{\Delta I_c}{I_c} + O\left(\left(\frac{\Delta I_c}{I_c}\right)^2\right)$$

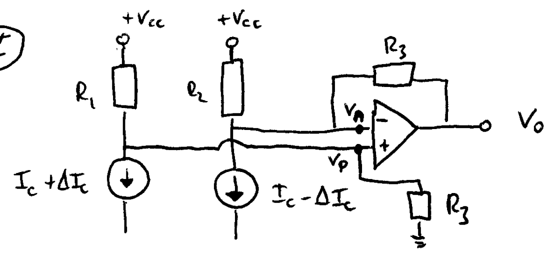
$$\frac{I_x + \Delta I_x}{I_x - \Delta I_x} = \frac{1 + (\Delta I_x / I_x)}{1 - \Delta I_x / I_x} = 1 + 2 \frac{\Delta I_x}{I_x} + O\left(\left(\frac{\Delta I_x}{I_x}\right)^2\right)$$

continue

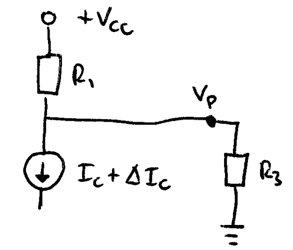
$$\frac{I_c + \Delta I_c}{I_c - \Delta I_c} = \frac{I_x + \Delta I_x}{I_x - \Delta I_x} \Rightarrow 1 + 2 \cdot \frac{\Delta I_c}{I_c} = 1 + 2 \frac{\Delta I_x}{I_x}$$

$$\Rightarrow \Delta I_c = I_c \cdot \frac{\Delta I_x}{I_x} = \frac{I_y}{2} \cdot \frac{\Delta I_x}{I_x} = \frac{V_y}{2 R_y} \cdot \frac{V_x}{R_x} \cdot \frac{1}{I_0}$$

IV

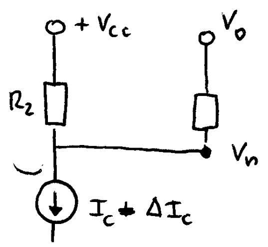


$$V_p = +V_{cc} - (I_c + \Delta I_c) \cdot (R_1 /\!/ R_3)$$



$$V_p = \frac{R_3}{R_1+R_3} \cdot V_{cc} - (I_c + \Delta I_c) \cdot \frac{R_1 R_3}{R_1 + R_3}$$

$$\ll g \gg$$



$$V_n = \frac{R_3}{R_2+R_3} V_{cc} + \frac{R_2}{R_2+R_3} V_o - (I_c - \Delta I_c) \frac{R_2 R_3}{R_2+R_3}$$

ideal opamp: $V_p = V_n$

$$V_o = \frac{R_2+R_3}{R_2} \cdot \left[\frac{R_3}{R_1+R_3} V_{cc} - \frac{R_3}{R_2+R_3} V_{cc} - (I_c + \Delta I_c) \frac{R_1 R_3}{R_1+R_3} + (I_c - \Delta I_c) \frac{R_2 R_3}{R_2+R_3} \right]$$

$$\ll f \gg$$

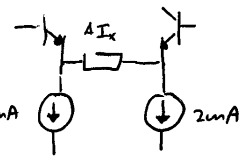
$R_1 = R_2 \Rightarrow$

$$V_o = -2 R_3 \Delta I_c$$

$$= - \frac{R_3}{R_x R_y} \cdot \frac{1}{I_o} V_x V_y \quad , \qquad \begin{aligned} R_3 &= 2\,k\Omega \\ R_x &= 1\,k\Omega \\ R_y &= 1k\,\Omega \\ I_o &= 2\,mA \end{aligned}$$

$$\boxed{V_o = - V_x V_y \quad [V]}$$

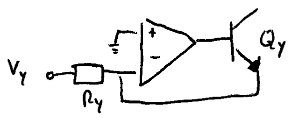
c)



$$\left.\begin{array}{l}|\Delta I_x|\ max = 2\,mA \\ R_x = 1\ K\Omega\end{array}\right\}|\Delta V_x|\ max = 2V$$

$\Rightarrow \quad |V_x| \leq 2\ V$

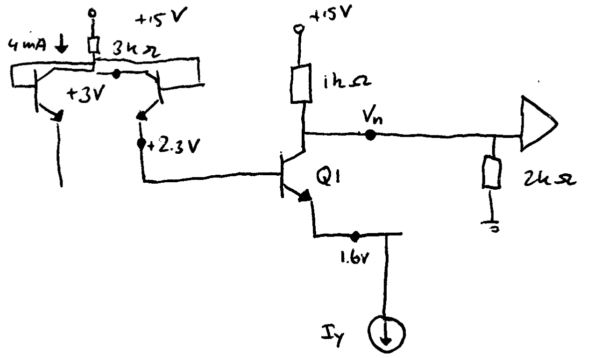
$(\ V_{bex_1} = V_{bex_2} = 0.7V\ )$

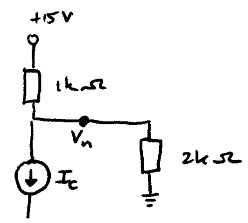


$V_{by} \geqslant V_{ey} \quad \Longrightarrow \quad V_y \leq 0$

(veja folha de problemas 1)

Para $V_y > 0$ o circuito fica em malha aberta e a saida do opamp satura em $-V_{cc}$. A tensão inversa elevada entre a base e o emissor de Qy pode danificar o transistor ; a função do diodo é limitar a tensão de polarização inversa de junção a aprox. $-0.7\,V$





$Q_1 \quad V_{bc}$ polarização inversa : $\quad V_n \geqslant 2.3\ V$

$$\Rightarrow \quad I_c \leqslant \frac{+15V - 2.3V}{1k\Omega} - \frac{2.3V}{2k\Omega} = 12.7\ mA - 1.15\ mA$$

$$= 11.55\ mA$$

$$\Rightarrow \quad I_y \leq 2\, I_c \leqslant 23.1\ mA$$

$$V_y = -R_y \cdot I_y \geqslant -23.1\ V \qquad (V_x = 0V)$$

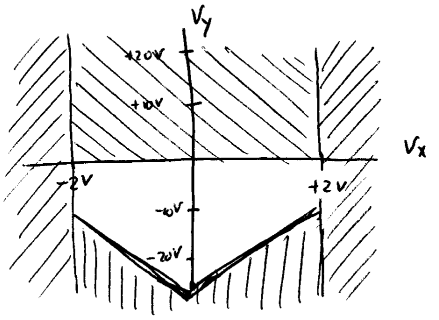
Quando $\quad V_x = V_{x\,max} = +2V$: $\quad \Delta I_c = I_c \cdot \dfrac{\Delta I_x}{I_0}$

$$\Delta I_c = I_c \cdot \frac{\Delta I_x}{I_0} = \frac{I_c}{I_0} \cdot \frac{V_x}{R_x} = I_c \cdot \frac{2V}{2mA \cdot 1k\Omega} = I_c$$

$$I_c' = I_c + \Delta I_c = 2 I_c$$

$$I_y' = I_c'/2 = I_c \quad , \quad I_y'^{\,max} = I_c^{max} = 11.55\ mA$$

$$V_y'^{\,min} = -R_y I_c^{max} = -11.55\ V$$



$\Leftarrow$ 2 quadrantes

d) $I_{S2} = I_{S1}(1+\delta)$

volta a «d» da pagina 2

$$I_{c1} = I_{S1} \exp\left(\frac{V_{b1}-V_{e1}}{V_T}\right) \equiv I_c + \Delta I_c$$

$$I_{c2} = I_{S2} \exp\left(\frac{V_{b2}-V_{e2}}{V_T}\right) \equiv I_c - \Delta I_c$$

$$\frac{I_c + \Delta I_c}{I_c - \Delta I_c} = \frac{I_{c1}}{I_{c2}} = \frac{I_{S1}}{I_{S2}} \cdot \exp\left(\frac{V_T}{V_T} \cdot \log\left(\frac{I_x + \Delta I_x}{I_x - \Delta I_x}\right)\right)$$

Taylor :

$$\left(1 + 2\frac{\Delta I_c}{I_c}\right) = \frac{I_{S1}}{I_{S2}}\left(1 + 2\frac{\Delta I_x}{I_x}\right) \Rightarrow$$

$$\frac{\Delta I_c}{I_c} = \frac{1}{2}\left(\frac{I_{S1}}{I_{S2}} - 1\right) + \frac{\Delta I_x}{I_x} \Rightarrow \Delta I_c = \frac{I_c}{2}\left(\frac{I_{S1}}{I_{S2}} - 1\right) + I_c \cdot \frac{\Delta I_x}{I_x}$$

$$I_{S1} = (1+\delta) I_{S2} \Rightarrow$$

$$\Delta I_c = \frac{\delta}{2} I_c + I_c \cdot \frac{\Delta I_x}{I_x} \Rightarrow V_o = -2R_3 \Delta I_c = -\delta R_3 I_c - 2R_3 I_c \, \Delta I_x / I_x$$

$$I_c = \frac{I_y}{2} = \frac{V_y}{2R_y} \quad , \quad \Delta I_x = \frac{V_x}{R_x} \quad , \quad I_x = I_o$$

$$\Rightarrow V_o = -\delta R_3 \frac{V_y}{2R_y} - 2 R_3 \frac{I_y}{2} \cdot \frac{V_x}{R_x} \cdot \frac{1}{I_o}$$

erro $(R_3 = 2R_y = 1k\Omega)$

$$\Delta V_o = -\delta V_y$$

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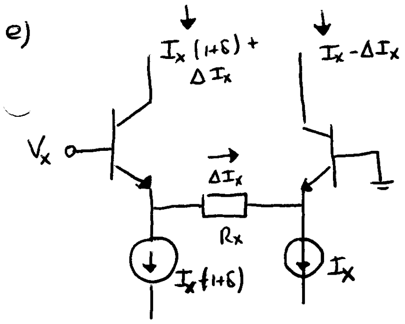
e)



$$V_{bex1} = V_{bex2} = 0.7\,V$$

$$\Delta I_x = \frac{V_{ex1} - V_{ex2}}{R_x} = \frac{V_x}{R_x}$$

em (II) : $$V_{be3} - V_{be4} = V_T \log\left(\frac{I_x + \delta I_x + \Delta I_x}{I_x - \Delta I_x}\right)$$

em (III) : $$\frac{I_c + \Delta I_c}{I_c - \Delta I_c} = \frac{I_x + \delta \cdot I_x + \Delta I_x}{I_x - \Delta I_x}$$

$$\left(1 + 2\frac{\Delta I_c}{I_c}\right) = \left(1 + 2\frac{\Delta I_x}{I_x}\right) + \frac{\delta \cdot I_x}{(I_x - \Delta I_x)}$$

$$\Delta I_c = I_c \cdot \frac{\Delta I_x}{I_x} + \frac{1}{2} \cdot I_c \cdot \frac{\delta \cdot I_x}{(I_x - \Delta I_x)}$$

em (IV)

$$V_o = -2 R_3 \Delta I_c \quad \Rightarrow \quad \text{erro} =$$

$$\Delta V_o = -\cancel{4}\, R_3 \cdot I_c \cdot \frac{\delta \cdot I_x}{(I_x - \Delta I_x)}$$

$$= -V_y \cdot \delta \cdot \frac{2\,mA}{2\,mA - V_x\,(mA)}$$

$R_y = 1k\Omega$

$R_3 = 2k\Omega$

$I_c = \dfrac{V_y}{2R_y} = \dfrac{V_y\,(mA)}{2}$

$\Delta I_x = \dfrac{V_x}{R_x} = V_x \,(mA)$

$I_x = 2\,mA$

f) volta a pagina ④

$$V_o = \frac{R_2+R_3}{R_2}\left[\frac{R_3}{R_1+R_3}\cdot V_{cc} - \frac{R_3}{R_2+R_3} V_{cc} - (I_c + \Delta I_c)\frac{R_1 R_3}{R_1+R_3} + \right.$$

$$\left. + (I_c - \Delta I_c)\frac{R_2 R_3}{R_2+R_3}\right]$$

$R_2 = R_1(1+\delta) \implies$

$$\frac{R_3}{R_2+R_3} = \frac{R_3}{R_1+R_3+\delta\cdot R_1} \overset{Taylor}{=\!=} \frac{R_3}{R_1+R_3} - \frac{R_3 R_1}{(R_1+R_3)^2}\cdot\delta$$

$$\frac{R_2+R_3}{R_2} = \frac{R_1+R_3+\delta R_1}{R_1+\delta\cdot R_1} \overset{Taylor}{=\!=} \frac{R_1+R_3}{R_1} + \frac{R_3}{R_1}\cdot\delta$$

$$\frac{R_2 R_3}{R_2+R_3} = \frac{R_1 R_3 + \delta R_1 R_3}{R_1+R_3+\delta\cdot R_1} \overset{Taylor}{=\!=} \frac{R_1 R_3}{R_1+R_3} + \frac{R_1 R_3^2}{(R_1+R_3)^2}\cdot\delta$$

$$V_o = \left(\frac{R_1+R_3}{R_1} + \frac{R_3}{R_1}\cdot\delta\right)\left[\frac{R_3 R_1}{(R_1+R_3)^2}\cdot\delta\cdot V_{cc} - \Delta I_c\cdot\frac{2R_1R_3}{R_1+R_3} - \Delta I_c\cdot\frac{R_1R_3^2}{(R_1+R_3)^2}\cdot\delta \right.$$

$$\left. + I_c\cdot\frac{R_1R_3^2}{(R_1+R_3)^2}\cdot\delta\right]$$

$$\Delta V_o = \left\{-\frac{R_3}{R_1}\cdot\frac{2R_1R_3}{R_1+R_3}\cdot\Delta I_c\cdot + \frac{R_1+R_3}{R_1}\left[\frac{R_3 R_1}{(R_1+R_3)^2}\cdot V_{cc} - \Delta I_c\frac{R_1R_3^2}{(R_1+R_3)^2}\right.\right.$$

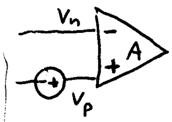
$$\left.\left. + I_c\cdot\frac{R_1R_3^2}{(R_1+R_3)^2}\right]\right\}\delta + O(\delta^2)$$

$$\Delta V_o = \delta\left\{-\frac{2R_3^2}{R_1+R_3}\cdot\Delta I_c + \frac{R_3}{R_1+R_3}\cdot V_{cc} - \Delta I_c\frac{R_3^2}{R_1+R_3} + I_c\frac{R_3^2}{R_1+R_3}\right\}$$

$$= \delta\cdot\frac{R_3}{R_1+R_3}\left\{-\underbrace{3R_3\Delta I_c}_{\sim V_x V_y} + \underbrace{R_3 I_c}_{\sim V_y} + \underbrace{V_{cc}}_{\sim 1}\right\}$$

g) volta a <<g>> , pagina 4

$$V_p = \frac{R_3}{R_1+R_3} V_{cc} - (I_c + \Delta I_c) \frac{R_1 R_3}{R_1+R_3} + V_{off}$$



$$V_n = \frac{R_3}{R_1+R_3} V_{cc} - \frac{R_1}{R_1+R_3} V_o - (I_c - \Delta I_c) \frac{R_1 R_3}{R_2+R_3}$$

ideal opamp $\Rightarrow V_p = V_n$

$$-\Delta I_c \cdot \frac{R_1 R_3}{R_1+R_3} + V_{off} = -\frac{R_1}{R_1+R_3} V_o + \Delta I_c \cdot \frac{R_1 R_3}{R_2+R_3}$$

$$V_o = \frac{R_1+R_3}{R_1} \cdot \left[2 \Delta I_c \cdot \frac{R_1 R_3}{R_1+R_3} - V_{off} \right]$$

$$\Delta V_o = -\frac{R_1+R_3}{R_1} \cdot V_{off}$$