Komplements of Electronics

Exame 14/01/2013 (23/01/2013)

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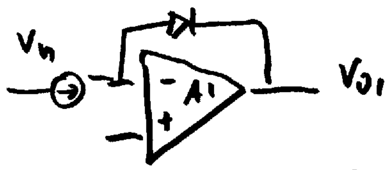
1 Ebers - Moll diode : $I = I_0 \left\{ \exp\left(\frac{\Delta V}{V_T}\right) - 1 \right\}$

Ideal opamp $V_p = V_n$ $\Delta V \approx V_T \ln\left(\frac{I}{I_0}\right)$

A1



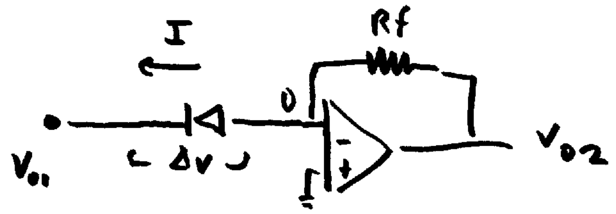
$$V_p = V_i \times \frac{R_1}{R_1 + R_2}$$

$$V_n = V_p = V_i \times \frac{R_1}{R_1 + R_2}$$

$$V_{01} = V_n - \Delta V$$

$$= + V_i \times \frac{R_1}{R_1 + R_2} - V_T \ln\left(\frac{I_{ref}}{I_0}\right)$$

A2



$$\Delta V = 0 - \left(+ V_i \times \frac{R_1}{R_1 + R_2} - V_T \ln\left(\frac{I_{ref}}{I_0}\right)\right)$$

$$I = I_0 \exp\left(\frac{\Delta V}{V_T}\right) = I_0 \exp\left[\frac{-V_i}{V_T} \frac{R_1}{R_1 + R_2} + \frac{V_T}{V_T} \ln\left(\frac{I_{ref}}{I_0}\right)\right]$$

$$= I_0 \exp\left(-\frac{V_i}{V_T} \times \frac{R_1}{R_1 + R_2}\right) \cdot \exp\left[\ln\left(\frac{I_{ref}}{I_0}\right)\right]$$

$$= I_0 \exp\left(-\frac{V_i}{V_T} \times \frac{R_1}{R_1 + R_2}\right) \cdot \frac{I_{ref}}{I_0}$$

$$V_{o2} = I \times R_f = R_f \cdot I_{ref} \cdot \exp\left(-\frac{V_i}{V_T} \cdot \frac{R_1}{R_1 + R_2}\right)$$

2

op amp is ideal

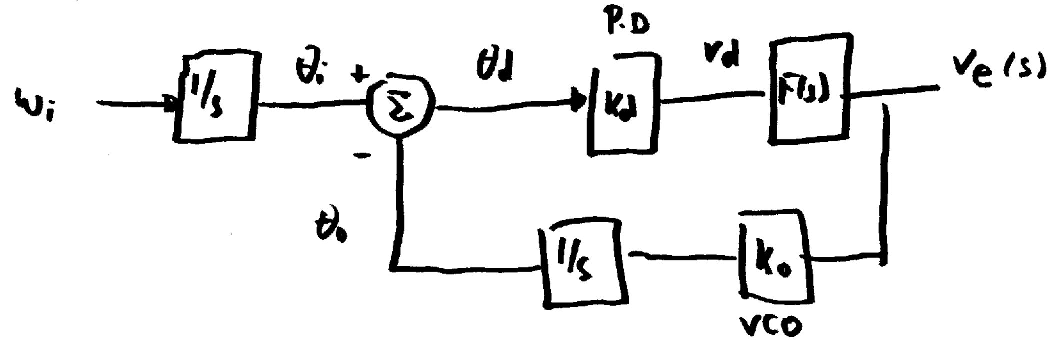
- no current to and from inputs!
- $V_n = V_p$

Thus

* $V_n = V_p = V_i$
* no current to input $\Rightarrow$ no current through R_1 $\Rightarrow$ V_3 (top of R_3) $= V_n = V_i$
* Therefore $I_3 = \frac{V_3 - 0}{R_3} = \frac{V_i}{R_3}$
* $\beta = \infty \Rightarrow I_C = I_E = I_3 = \frac{V_i}{R_3}$

$$\boxed{I = \frac{V_i}{R_3}}$$

3 a) See lecture notes



$$\theta_o = (\theta_i(s) - \theta_o(s))\, K_d\, F(s)\, K_o \cdot \frac{1}{s} \Rightarrow$$

$$H(s) \equiv \frac{\theta_o(s)}{\theta_i(s)} = \frac{K_v F(s)}{s + K_v F(s)}, \text{ with } K_v = K_d K_o$$

b) without LPF

$$H(s) = \frac{K_v}{K_v + s} = \frac{1}{1 + s/K_v} \qquad (s = j\omega)$$

compares with behavior of a low-pass filter

$$\frac{1}{1 + j\omega/\omega_c}$$ with cut-off frequency

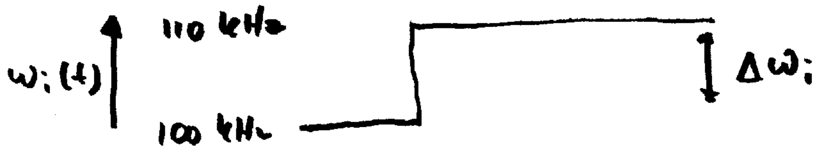
$$\omega_c = K_v = K_d K_o = 1 \frac{V}{rad} \times 2 \frac{kHz}{V}$$

$2\ kHz = 2000 \times 2\pi\ rad/s \Rightarrow$

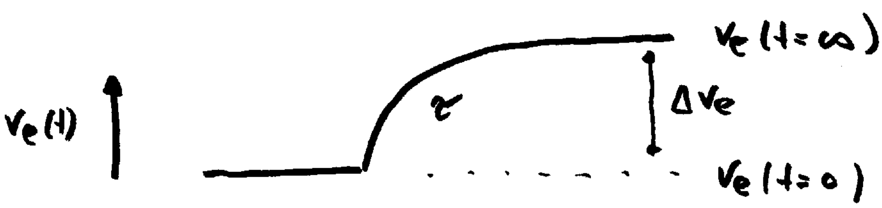
$$\omega_c = 2000 \times 2\pi \ \frac{\cancel{rad}}{s \cdot \cancel{V}} \times \frac{\cancel{V}}{\cancel{rad}} = 4000\pi\ Hz$$

c) $$\frac{V_e(s)}{\omega_i(s)} = \frac{1}{K_o} H(s),$$ because $\theta_i(s) = \frac{\omega_i(s)}{s}$, and $\theta_o(s) = \frac{v_e K_o}{s}$

$$= \frac{1/K_o}{1 + s/K_v}$$ } again, like a LPF! (with DC gain $1/K_o$)

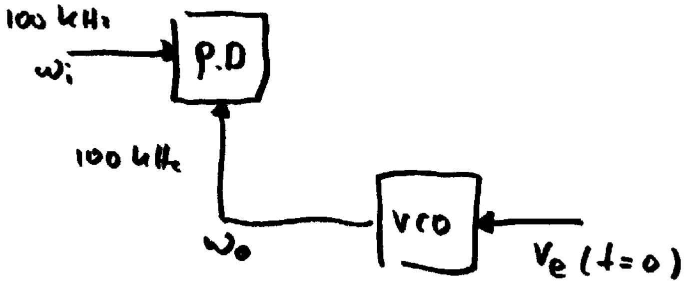


From circuit analysis, we know that the response of a LPF to a step function in time is a exponential aproximation



τ = 'RC' = $1/\omega_c$ = $1/K_v$ = $\frac{1}{4000\pi}$ s

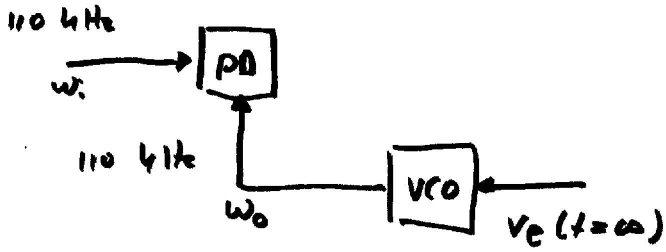
$v_e(t=0)$: given the fact that $\omega_o = \omega_i$ (we have lock), the PLL looks like this:



VCO : ω_o = 100 kHz + $\frac{2\text{ kHz}}{V} \times V_e$ = 100 kHz

thus $V_e = 0$

$v_e(t=\infty)$: we again have lock



ω_o = 100 kHz + $\frac{2\text{ kHz}}{V} \times V_e$ = 110 kHz

thus V_e = 5 V

Δv_e can also be found by using the DC gain of the filter-like behavior,

$\Delta V_e = \frac{1}{K_o} \times \Delta\omega_i = \frac{1}{2}\frac{V}{\text{kHz}} \times 10$ kHz

= 5 V

d) From electronics II we know about stability of feedback systems. For negative feedback systems like ours (note the $\overset{+}{\underset{-1}{\Sigma}}$ part) the transfer function was

$$\frac{V_o}{V_i} = \frac{A}{1+A\beta}$$

and this meant analyzing $A\beta$ and see if it was equal to -1 (B.C.) or smaller, etc.

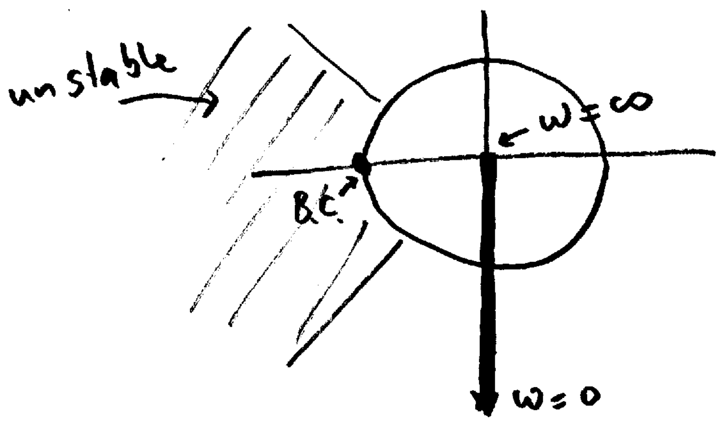
$A\beta$ in our case is

$$A\beta = K_d\, F(s)\, K_o \cdot \frac{1}{s}$$

without filter:

$$A\beta = K_d K_o / s$$

$$= \frac{K_d K_o}{j\omega}$$



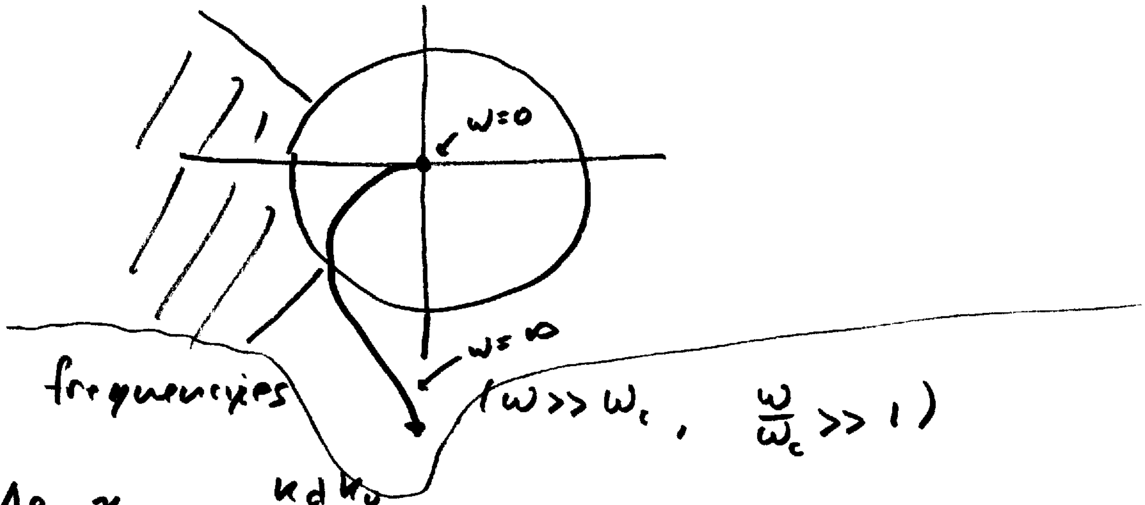
stable!

Now add a low-pass filter

$$A\beta = \frac{K_d K_o}{s\,(1 + s/\omega_c)} = \frac{K_d K_o}{j\omega\,(1 + j\omega/\omega_c)}$$

with $\omega_c = \frac{1}{\tau} = \frac{1}{RC} = \frac{1}{10^{-6}\,s}$

$$A\beta = \frac{K_d K_o}{(j\omega - \omega^2/\omega_c)}$$



High frequencyies ($\omega \gg \omega_c$, $\frac{\omega}{\omega_c} \gg 1$)

$$A\beta \approx \frac{K_d K_o}{-\omega^2/\omega_c} = -0$$

(real and negative)
($\phi = 180°$)

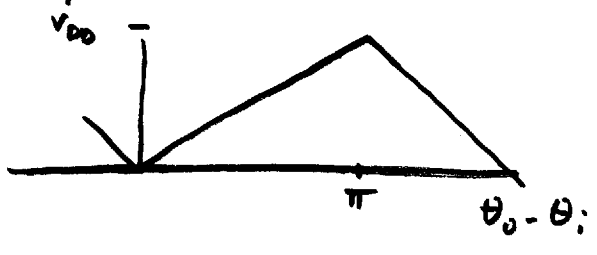
Low frequencies ($\omega \ll \omega_c$, $\frac{\omega}{\omega_c} \ll 1$, $1 + j\frac{\omega}{\omega_c} \approx 1$)

$$A\beta \approx \frac{K_d K_o}{j\omega} = -j\,\infty \quad (\phi = -90°)$$

This is potentially not stable, since it can enter the danger zone. Yet, too complicated to analytically solve in this exam. Make a matlab simulation.

4] see lecture notes

P.D type I is a XOR port



$$K_D = \frac{V_{DD}}{\pi}$$