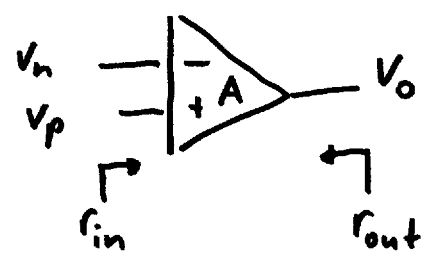
(1) Logarithmic and antilogarithmic (exponential) amplifiers

These amplifiers are based on the operational amplifiers (opamps) of our earlier lectures. Remember, the ideal operational ampl.:



$$V_0 = A \cdot (V_p - V_n)$$

1) $r_{in} = \infty$: no current ever enters into inputs $-$ and $+$

2) $r_{out} = 0$: output is ideal voltage source. It can supply any current needed to maintain 'programmed' V_0

3) $\Delta f = \infty$: infinite bandwidth, $A(f) = A$ for all frequencies.

4) $A = \infty$: infinite gain. And this means there are only two possible situations:

(4a) $V_o = \pm\infty$, saturation. If $V_p \neq V_n$, the output saturates at supply voltages

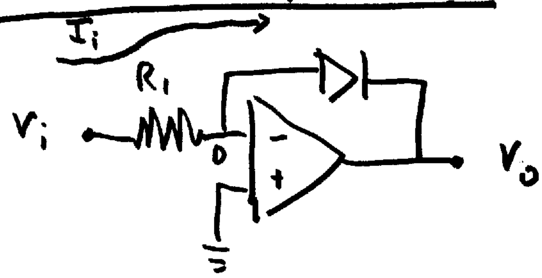
or

(4b) $V_p = V_n$, $V_o = A \cdot (V_p - V_n)$

$$V_p - V_n = \frac{V_o}{A}, \quad A = \infty \Rightarrow$$

$$V_p - V_n = 0$$

Logarithmic amplifier



V_p is at ground, thus V_n is virtual ground (4b). The input current through R_1 is then

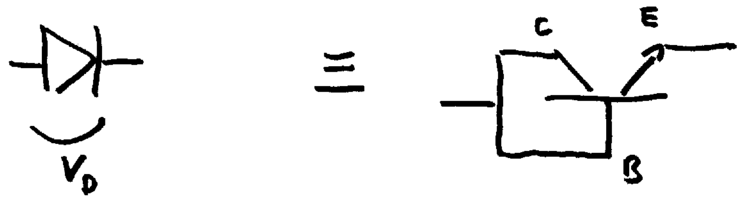
$$I_i = \frac{V_i - \emptyset}{R_1} = \frac{V_i}{R_1}$$

This current cannot enter the opamp (1: $r_{in} = \infty$) and thus must go through the diode.

Ebers Moll of transistor (bipolar)

$$\boxed{I_E = I_0\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] + I_1\left[\exp\left(\frac{V_{CB}}{V_T}\right) - 1\right]}$$

A diode is equal to a transistor with the collector shorted to the base



$V_{CB} = 0$

$$I_D = I_o \left[\exp\left(\frac{V_D}{V_T} \right) - 1 \right]$$

If we ignore term '-1' we get

$$I_D = I_o \exp\left(\frac{V_D}{V_T} \right)$$

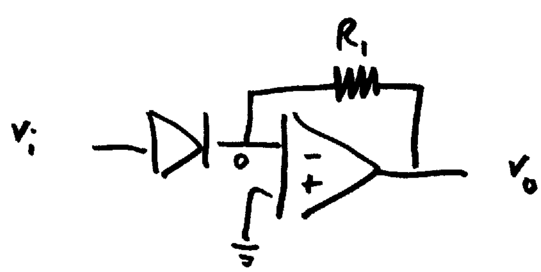
In our logarithmic amplifier $I_D = I_i = \frac{V_i}{R_1}$

Moreover, $V_o = 0 - V_D$

$$\frac{V_i}{R_1} = I_o \exp\left(\frac{V_D}{V_T} \right) \Rightarrow V_D = V_T \ln\left(\frac{V_i}{R_1 I_o} \right)$$

$$\boxed{V_o = -V_D = -V_T \ln\left(\frac{V_i}{R_1 I_o} \right)}$$

- <u>Exponential (antilogarithmic) amplifier</u>



V_p is at ground. V_n is at virtual ground.

The current through the diode is

$$I_i = I_0 \exp\left(\frac{V_i}{V_T}\right)$$

This current cannot enter the opamp and is forced through R_1, inducing a voltage drop

$$\Delta V = I_i \times R_1$$

Thus, $V_0 = 0 - \Delta V$,

$$V_0 = -R_1 I_0 \exp\left(\frac{V_i}{V_T}\right)$$

- Non-ideal effects of logarithmic amplifier
  - "-1" term
  - I_b polarizing currents of opamp
  - I_0 is unreliable (depends on diode used)
  - V_T depends on temperature

"-1" term. Instead of exponential function we should use full Ebers-Moll diode equation

$$I_D = I_0 \left[\exp\left(\frac{V_D}{V_T}\right) - 1\right]$$

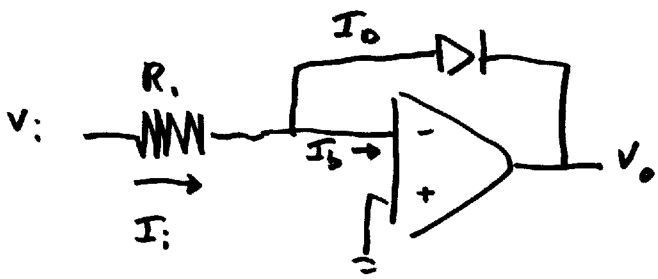
when does this become important? when is 1 comparable or larger than $\exp(V_D/V_T)$?

Obviously for voltages $V_D \leq 0$. For voltages $V_D \geq V_T \ln(20)$ the -1 is equal or less than

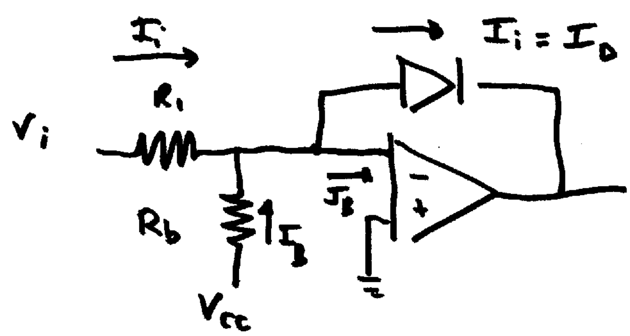
5% of $\exp\left(\frac{V_0}{V_T}\right)$ term. That is approximately 80 mV.

Looking back at out equation for V_0 at p. 3 it is obvious that it does not have a solution for $V_i < 0$.

I_b polarizing As we have seen in Electronics II, differential pairs, at the entrance of an opamp we find bipolar transistors. These are polarized by a current I_b. Tiny as it is ($\sim$100 nA), they change the log. amplifier



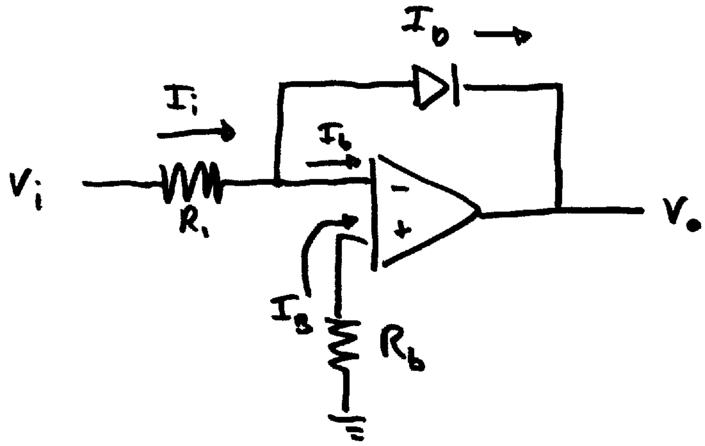
Because a part of I_i is lost in the form of current I_b, I_D is no longer equal to I_i, and the relation V_i and V_0 is no longer as found before. A solution is to inject the 'missing' current



If $\frac{V_{cc}}{R_b} = I_b$

then $I_D = I_i$

Another solution (that does not need knowledge of I_b!) is a resistor R_B at V_p



The current entering into + is equal to the one entering into − : I_b

In this case: $V_p = -R_b I_b$. The input current: $I_i = \frac{V_i - V_n}{R_1} = \frac{V_i - (-R_b I_b)}{R_1}$

From this, I_b is lost, so the diode current is

$$I_D = I_i - I_b = \frac{V_i + R_b I_b}{R_1} - I_b$$

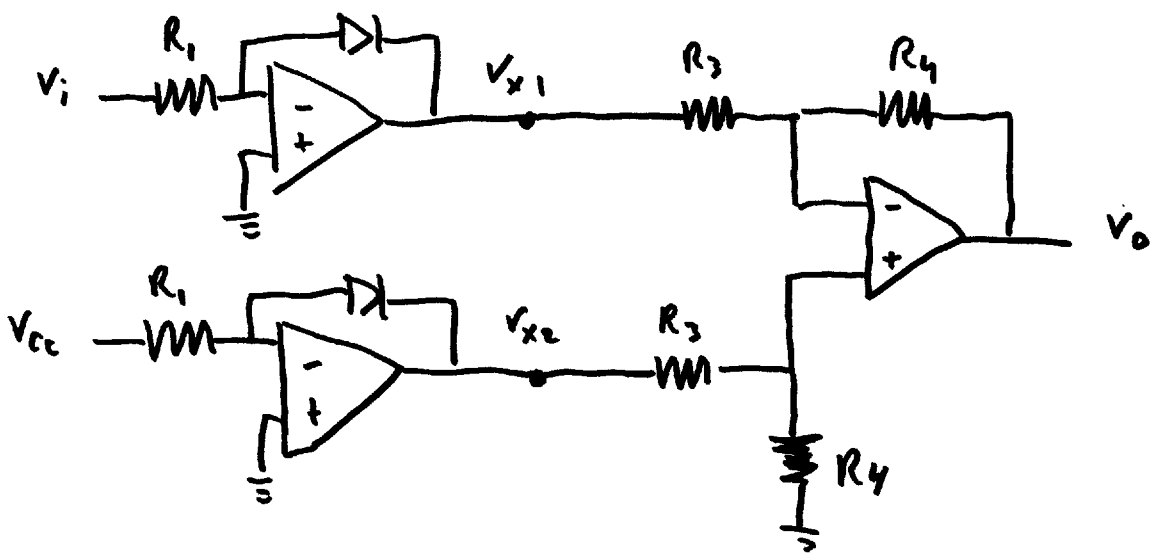
If $R_b = R_1$, then

$$I_D = \frac{V_i}{R_1}$$, as desired, independent of I_b!

I_0 (in)dependence All types of diodes have different I_0. This is unwanted, when our circuit depends on the type of diode we use. To circumvent this, we can make use of the mathematical property

$$\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$$

The circuit below shows how



$$V_{x1} = V_T \ln\left(\frac{V_i}{R_1 I_o}\right)$$

$$V_{x2} = V_T \ln\left(\frac{V_{cc}}{R_1 I_o}\right)$$

$$V_o = \frac{R_4}{R_3}\left(V_{x2} - V_{x1}\right) = -\frac{R_4}{R_3} V_T \ln\left(\frac{V_i}{V_{cc}}\right)$$

note the absence of I_o.

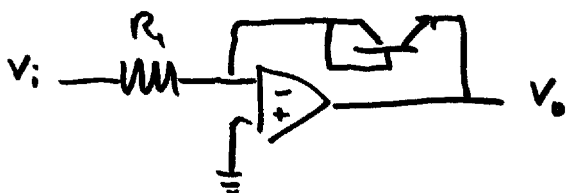
T independence In the above equation, $V_T = {}^{KT}/_{q}$, depends on temperature. We can eliminate the T dependence

$$\frac{dV_o}{dT} = -\frac{k}{q} \ln\left(\frac{V_i}{V_T}\right) \cdot \frac{d}{dT}\left(\frac{R_4(T)}{R_3(T)} \cdot T\right)$$

By selecting adequate resistors the dependence can be zero. Example. $R_4(T) = R_{4\emptyset}$ and $R_3(T) = R_{3\emptyset} T$.

Alternative configurations

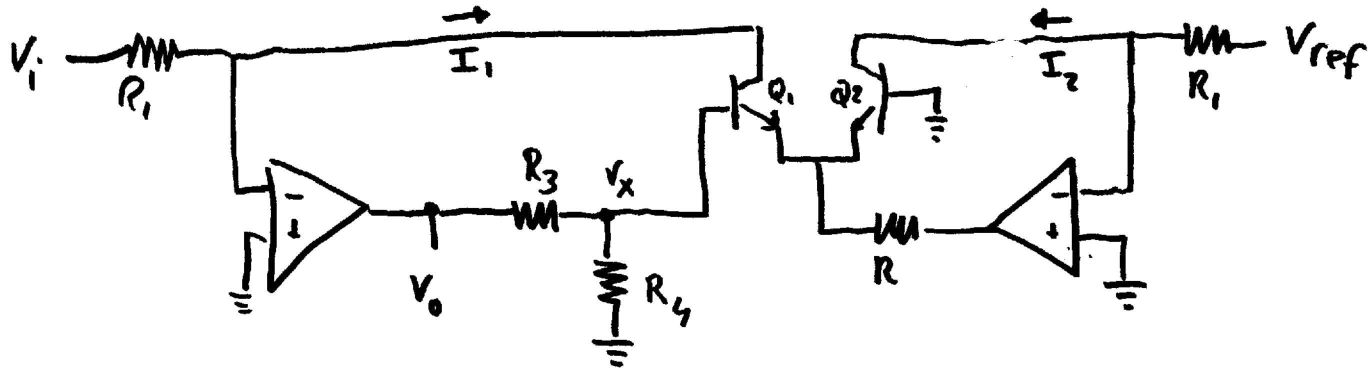
A diode is a transistor with C and B shorted; thus an alternative configuration is



Another one is with the base to ground

Since $V_B = V_n = 0$ (ground and virtual ground respectively), theoretically it is the same.

A configuration saving 1 opamp



starting at the base of transistor Q2 ($V_{B2}=0$) we see

$$V_x = -V_{BE2} + V_{BE1}$$

if r_{in2} is high (entrance resistance of Q_1)

$$V_x = V_0 \cdot \frac{R_4}{R_3+R_1} \Rightarrow V_0 = \frac{R_4+R_3}{R_4} V_x$$

Moreover,

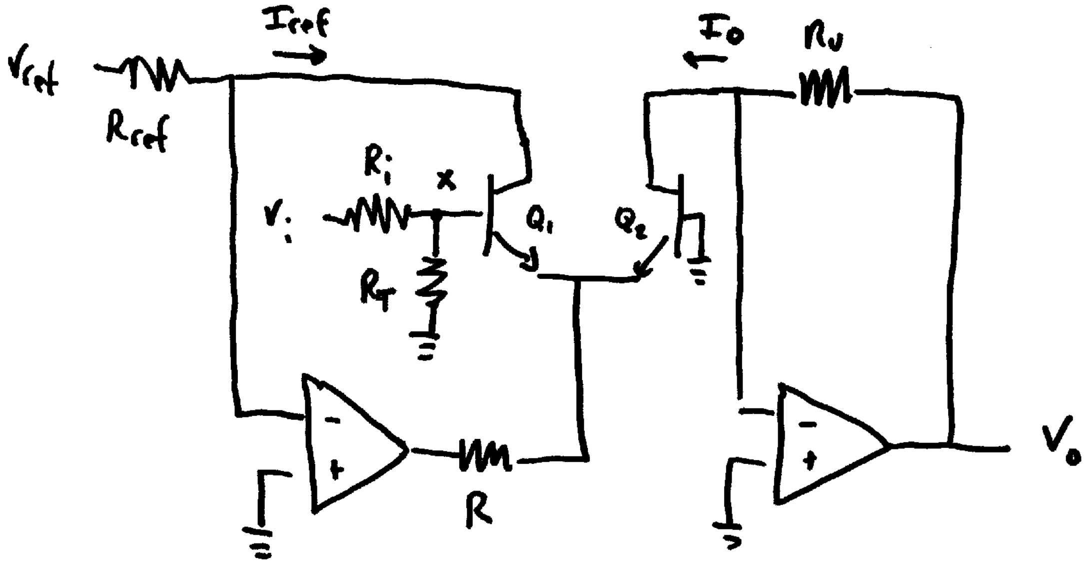
$$V_{BE1} = V_T \ln\left(\frac{I_1}{I_0}\right) = V_T \ln\left(\frac{V_i}{R_1 I_0}\right)$$

$$V_{BE2} = V_T \ln\left(\frac{I_2}{I_0}\right) = V_T \ln\left(\frac{V_{ref}}{R_1 I_0}\right)$$

Then

$$V_0 = \frac{R_4+R_3}{R_4} \times V_T \ln\left(\frac{V_i}{V_0}\right)$$

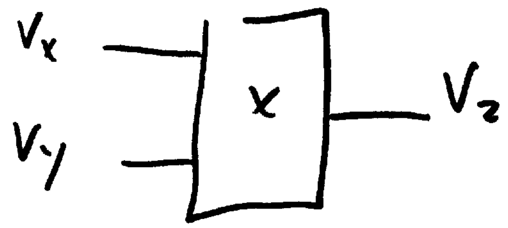
Exponential amplifier:



$$\begin{cases} V_x = 0 - V_{BE2} + V_{BE1} \\ I_{REF} = I_{S0} \exp\left(\frac{V_{BE1}}{V_T}\right) \Rightarrow V_{BE1} = V_T \ln\left(\frac{I_{ref}}{I_{S0}}\right) \\ I_o = I_{S0} \exp\left(\frac{V_{BE2}}{V_T}\right) \Rightarrow V_{BE2} = V_T \ln\left(\frac{I_o}{I_{S0}}\right) \\ V_x = V_i \times \frac{R_T}{R_T + R_i} \end{cases}$$

$$V_o = V_{ref} \times \frac{R_0}{R_{ref}} \times \exp\left(-\frac{V_i}{V_T} \cdot \frac{R_T}{R_T + R_i}\right)$$

Analog Multipliers



Analog multipliers have two analog voltages V_x and V_y at their input and the output voltage V_z is determined by

$$V_z = V_x \cdot V_y \cdot a$$

with a a constant.

An important parameter of analog multipliers is the allowed voltage range. Looking only at the sign of the allowed input voltages V_x and V_y, we can classify the multipliers as

single quadrant : V_x and V_y can be only positive (or only negative)

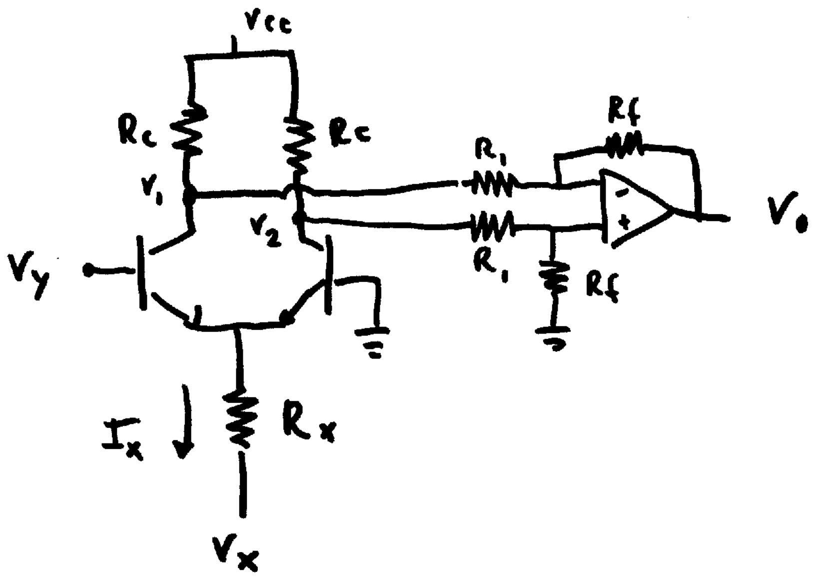
double quadrant : V_x or V_y can take both signs, but not both V_x and V_y

four quadrant : Both V_x and V_y can take both positive and negative values

Types of analog multipliers

- Transconductance
- log-antilog
- time-amplitude

A transconductance ~~ampt.~~ multiplier is given below. It consists basically of a differential pair amplifier (input V_y and 0), and a current source made of a voltage source (V_x) and a resistor



As we have seen from Electronics II,

$$V_1 = -\frac{V_y \cdot R_c}{2r_e}, \qquad V_2 = +\frac{V_y R_c}{2r_e}$$

This fed into a differential opamp:

$$V_o = \frac{R_f}{R_1} \cdot (V_2 - V_1) = \frac{R_f}{R_1} V_y \cdot \frac{R_c}{r_e} \qquad (\mathrm{I})$$

Moreover, the parameter r_e is given by the 'polarization' current $I_{E1}, I_{E2} = I_x/2$

$$r_e \equiv \frac{V_T}{I_{E1}} = \frac{V_T}{-V_x/2R_x} = -R_x \cdot \frac{V_T}{2V_x}$$

substituting in (I) of previous page,

$$V_o = -\frac{R_f}{R_1} \cdot V_y \cdot \frac{R_c}{R_x \cdot \frac{V_T}{2V_x}} = -\frac{2 R_f R_c}{R_1 R_x V_T} \cdot V_x V_y$$

Notes :

(A)
* The current I_x is not equal to $-V_x/R_x$ instead, $I_x = (-0.7 - V_x)/R_x$. This introduces a non-linearity in the output

(B)
* It is obvious that the transistors have to stay open, and thus

$$V_x \leqslant 0 \quad , \quad V_y \geqslant 0$$

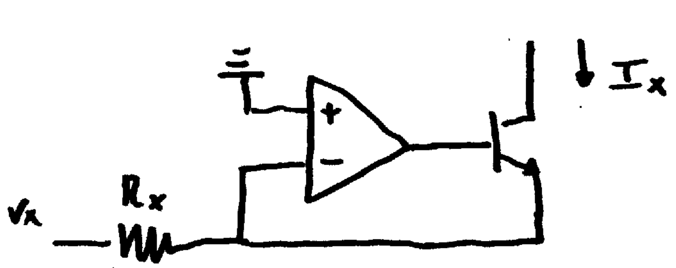
single quadrant !

(C)
* The circuit only works for small signals. Remember Electronics II. Linear for

$$V_y \leqslant 3V_T$$

A better I_x current source is

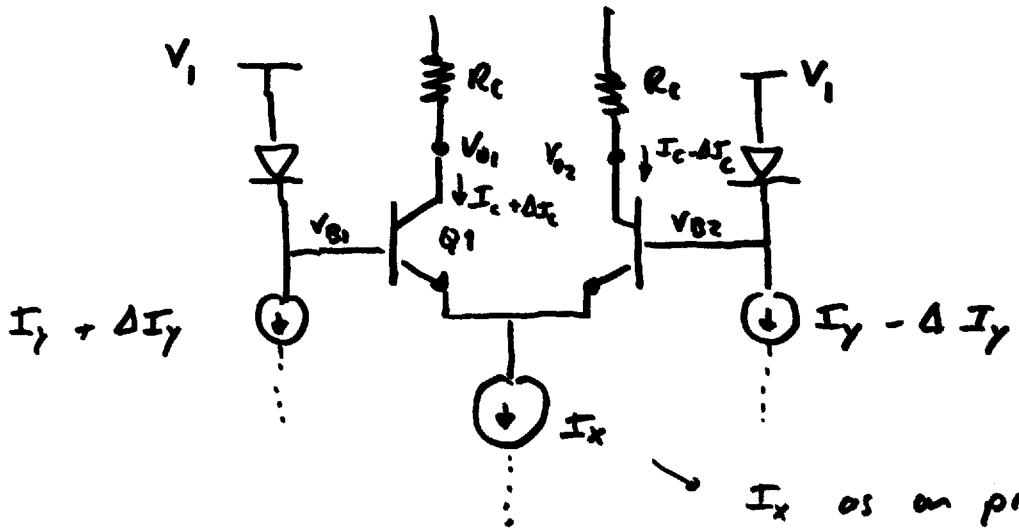
A:



This does not have the '0.7 V' effect.

$$I_x = \frac{\alpha V_x}{R_x}$$

$\boxed{c:}$ To make V_y range also large (instead of only until $3V_T$). we can make use of the following linearization circuit :



I_x as on previous page.

(Fig 13.8 Sergio Franco)

$$V_{B1} = V_1 - V_T \ln\left(\frac{I_y+\Delta I_y}{I_{S\phi}}\right)$$

$$V_{B2} = V_1 - V_T \ln\left(\frac{I_y-\Delta I_y}{I_{S0}}\right)$$

$$V_{B1}-V_{B2} = V_T \ln\left(\frac{I_y-\Delta I_y}{I_y+\Delta I_y}\right), = (V_{B1}-V_E)-(V_{B2}-V_E)$$

$$V_E = V_{E1} = V_{E2}$$

$$V_{BE1}-V_{BE2} = V_T \ln\left(\frac{I_y-\Delta I_y}{I_y+\Delta I_y}\right)$$

$$V_T \ln\left(\frac{I_c+\Delta I_c}{I_{S\phi}(\beta+1)}\right) - V_T \ln\left(\frac{I_c-\Delta I_c}{I_{S\phi}(\beta+1)}\right) = V_T \ln\left(\frac{I_y-\Delta I_y}{I_y+\Delta I_y}\right)$$

$$V_T \ln\left(\frac{I_c+\Delta I_c}{I_c-\Delta I_c}\right) = V_T \ln\left(\frac{I_y-\Delta I_y}{I_y+\Delta I_y}\right)$$

$$\frac{I_c+\Delta I_c}{I_c-\Delta I_c} = \frac{I_y-\Delta I_y}{I_y+\Delta I_y} \Rightarrow \frac{\Delta I_c}{I_c} = \frac{\Delta I_y}{I_y}$$

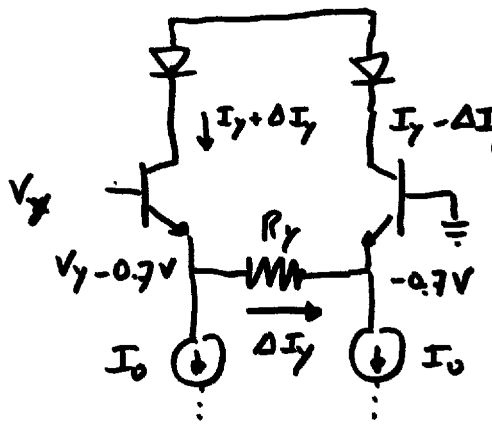
Moreover, $I_c = I_x / 2$

$$\Delta I_c = \frac{I_x \cdot \Delta I_y}{I_y} \qquad (\text{II})$$

I_x, as shown before, is proportional to V_x (p. 13)

What we need now is: i) a way to make ΔI_y proportional to V_y, and ii) a way to make V_o proportional to ΔI_c

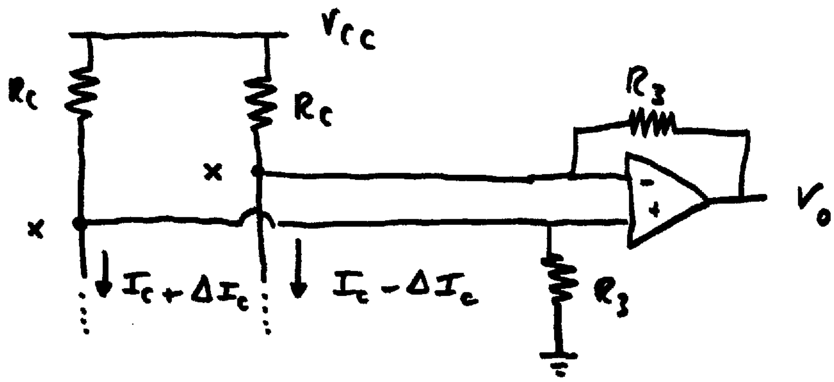
1)



$$\Delta I_y = V_y / R_y \qquad (A)$$

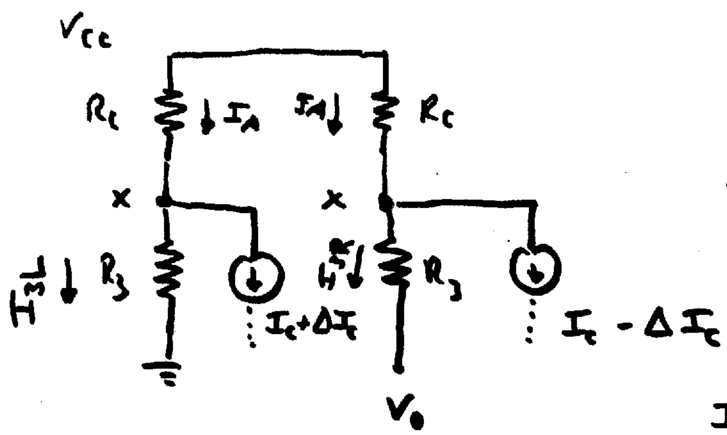
$$I_y = I_o$$

2)



$V_p = V_n$

$x = X$



$$I_{3L} = \frac{V_x}{R_3}, \quad I_{3R} = \frac{V_x - V_o}{R_3}$$

$$I_{3L} - I_{3R} = [I_A - (I_c + \Delta I_c)] - [I_A - (I - \Delta I_c)]$$

$$= -2\Delta I_c$$

$$I_{3L} - I_{3R} = \frac{V_x}{R_3} - \left(\frac{V_x - V_o}{R_3}\right) = \frac{V_o}{R_3}$$

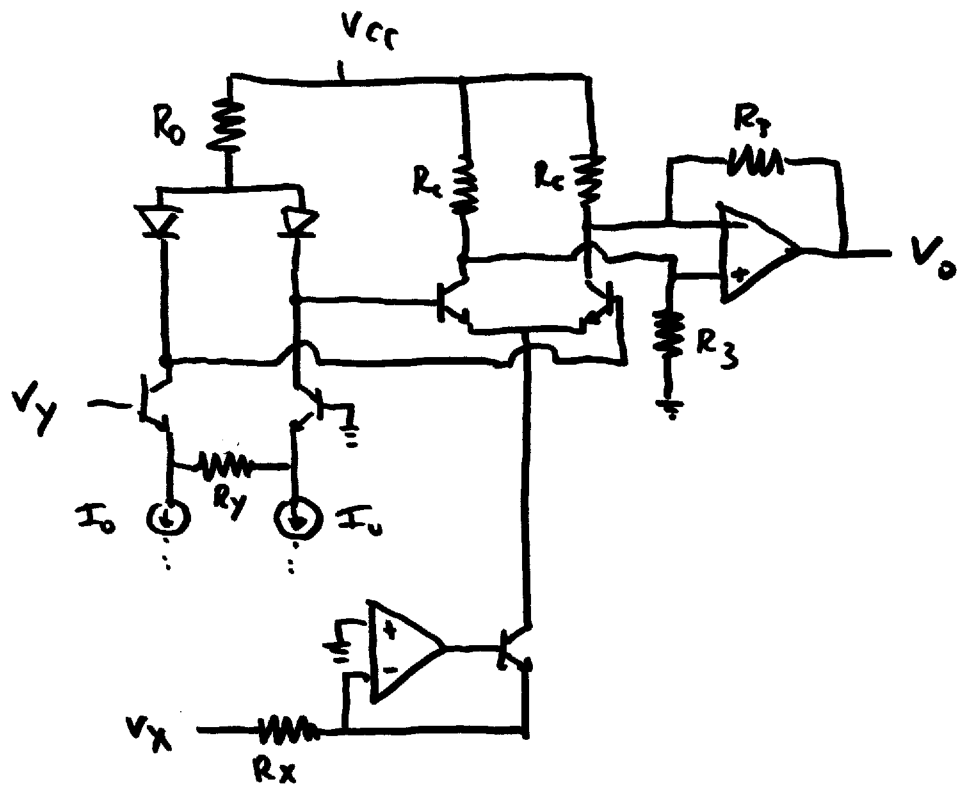
$$\frac{V_o}{R_3} = -2\Delta I_c \qquad (B)$$

(A) and (B) substituted into (II):

$$\frac{V_o}{2R_3} = \frac{\alpha V_x}{R_x} \cdot \frac{V_y}{R_y} \cdot \frac{1}{I_o} \quad \Rightarrow$$

$$\boxed{V_o = \frac{2 R_3 \alpha}{R_x R_y I_o} V_x V_y}$$

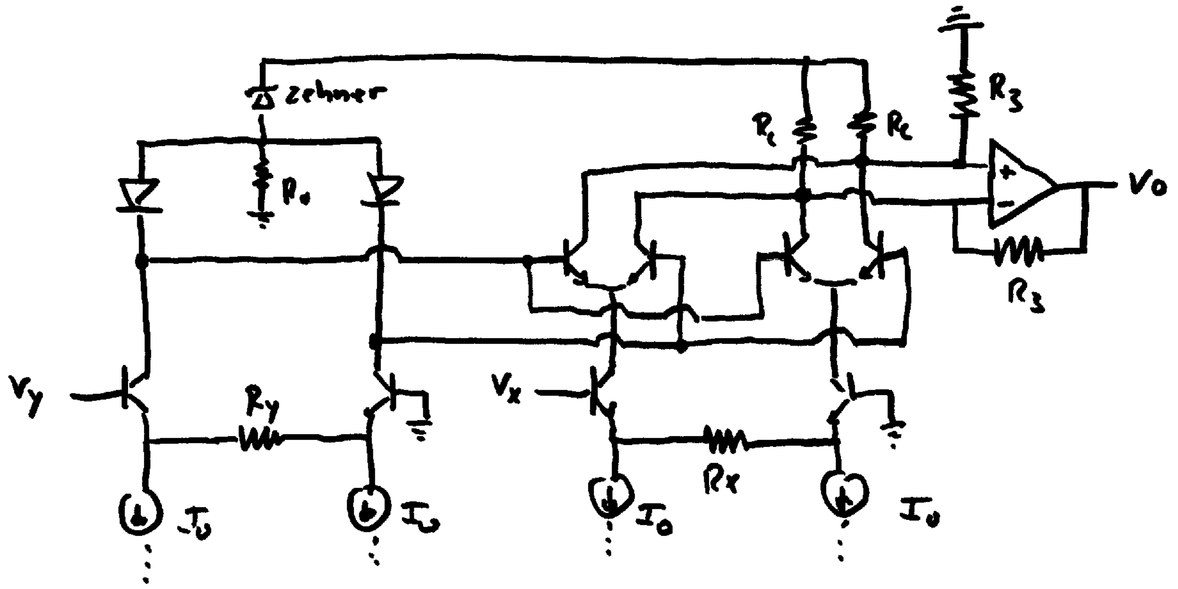
TOTAL CIRCUIT



* Vy can now be negative without problem!
  Two quadrants

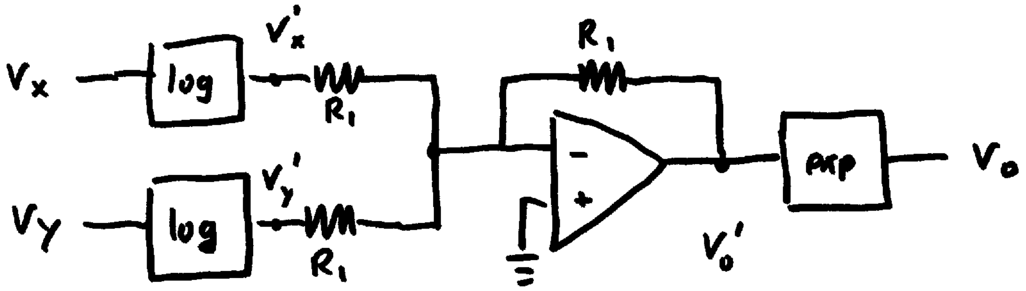
* Io is a current source / current mirror.
  Do you still remember how to make them?

4- quadrant multiplier. Let's repeat the trick for V_x :



4 - quadrant multiplier
(Fig 13.9 Sergio Franco)

Multipliers log - antilog



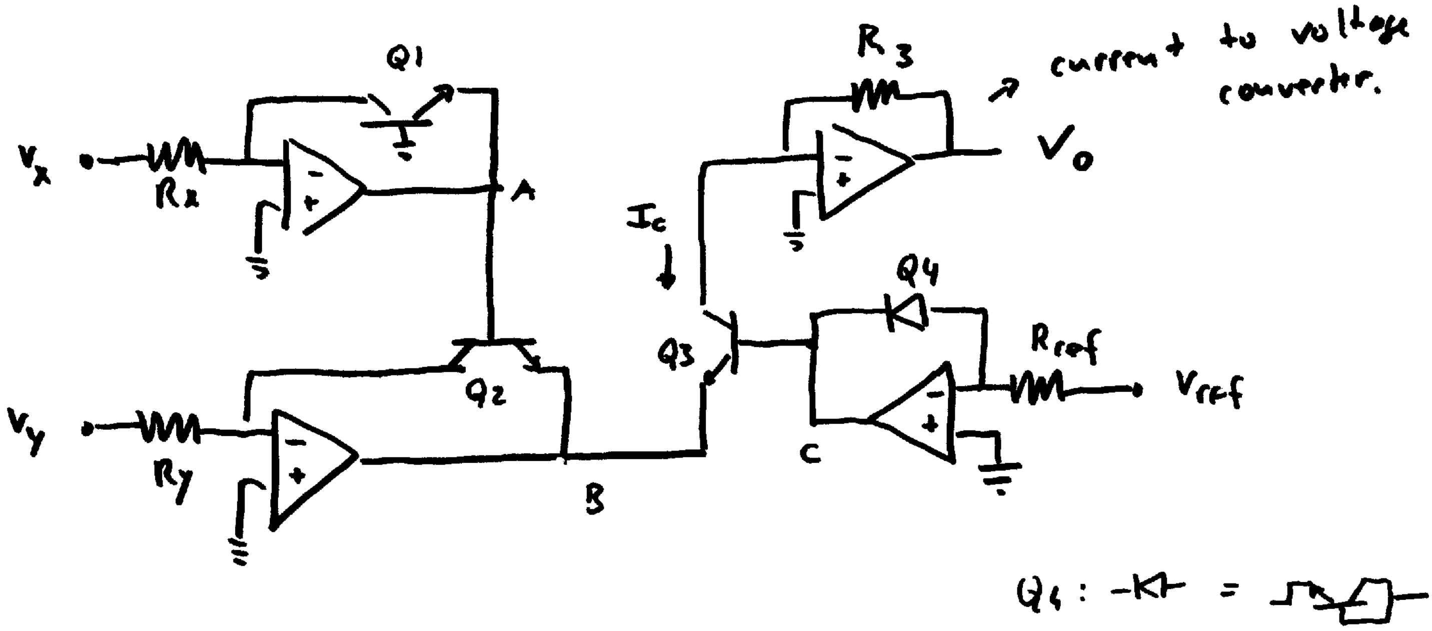
$$V_x' = \log(V_x) \quad , \quad V_y' = \log(V_y)$$

$$V_o' = \log(V_x) + \log(V_y)$$

$$V_o = \exp(V_o') = \exp[\ln(V_x) + \ln(V_y)]$$

$$= V_x V_y$$

The following circuit uses a 'trick' to do the summing of the two log signals :



The voltage at point A is

$$V_A = -V_T \ln\left(\frac{V_x}{R_x I_{S\phi}}\right)$$

$$V_B = V_A - V_T \ln\left(\frac{V_y}{R_y I_{S\phi}}\right) = -V_T \ln\left(V_x V_y / R_x R_y I_{S\phi}^2\right)$$

At point C

$$V_C = -V_T \ln\left(\frac{V_{ref}}{R_{ref} I_{S\phi}}\right)$$

$$V_{BE3} = V_C - V_B = V_T \ln\left(\frac{V_x V_y}{V_{ref}} \cdot \frac{R_{ref}}{R_x R_y I_{S\phi}}\right)$$

$$I_c = I_{S\phi} \exp\left(\frac{V_{BE3}}{V_T}\right) = \frac{V_x V_y}{V_{ref}} \cdot \frac{R_{ref}}{R_x R_y}$$

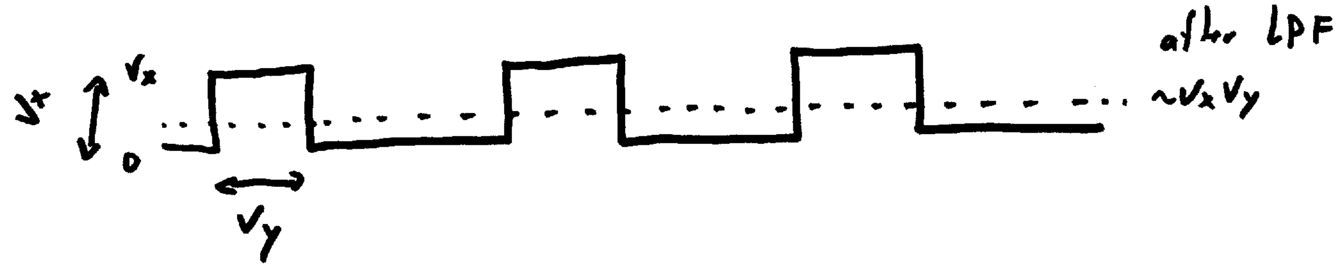
$$\boxed{V_o = 0 + I_c R_3 = \frac{R_{ref} R_3}{R_x R_y V_{ref}} \cdot V_x V_y}$$

I quadrant

$V_x \geq 0, V_y \geq 0$

Time-amplitude multipliers

The idea is that pulses are generated, with the height controlled by V_x and the width controlled by V_y. After low-pass filtering, the output is proportional to $V_x V_y$:



If the repetition frequency of the pulses is given by f, and if the 'on' time is given by αV_y, then, after LPF, the output is

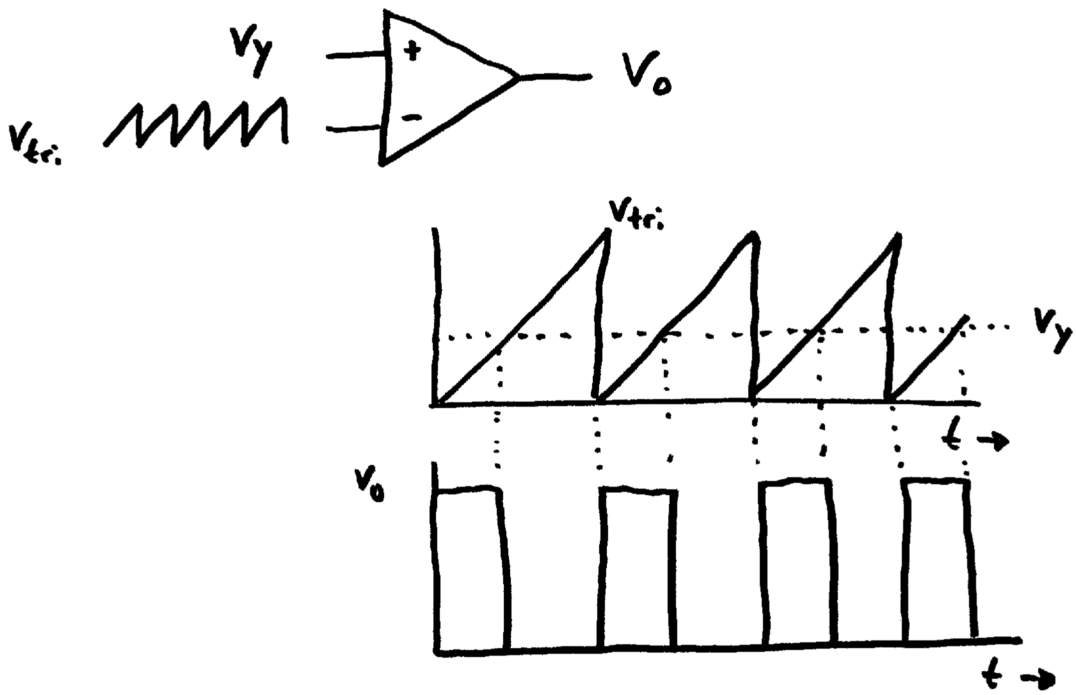
$$V_0 = \alpha f V_x V_y$$

Instead of pulse width or pulse height we can also use a variable repetition frequency,

f proportional to V_x

This can be done with voltage-controlled oscillators, VCOs

A simple voltage-to-(pulse) width converter is a comparator fed with a triangular reference wave in one input and the voltage at the other input

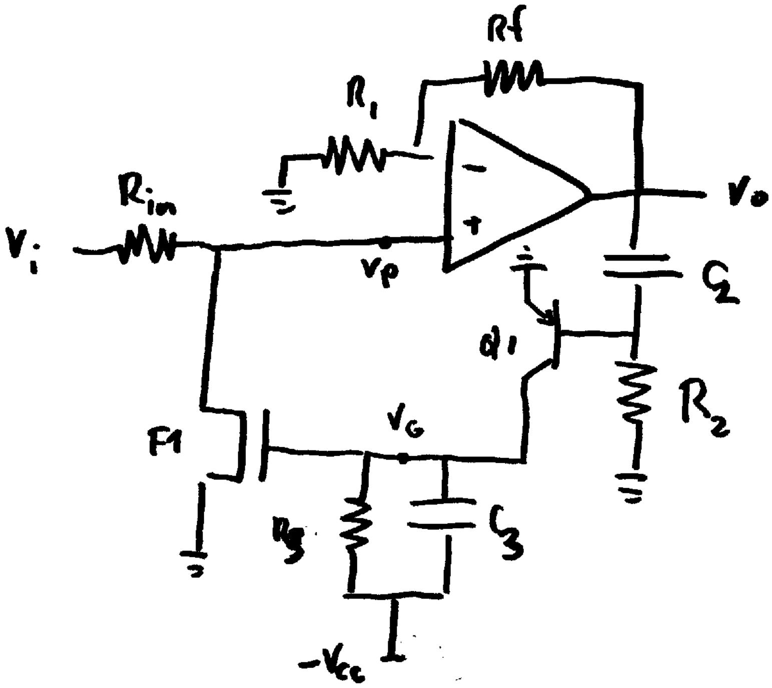


VCOs will be discussed later

AGC Automatic Gain Control

To avoid types of fading of signal (esp. AM signals). Microphones, etc.

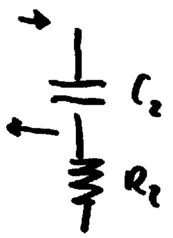
An example of an AGC amplifier is given here below:



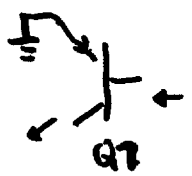
The following compounents can be recognized :

non - inverting amplifier

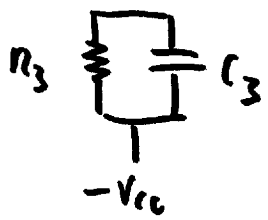
$$A = \frac{R_f + R_1}{R_1}$$



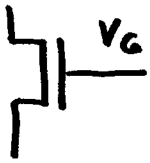
HPF (remove offset)



switch (opens for $V < -0.7$ V)



'LPF' . If Q1 opens, C_3 is discharged, if Q1 is closed, C_3 is charged through R_3



FET
(voltage-controled resistor)
output resistance $r_d \sim \frac{1}{V_G}$

The overal gain of the circuit is given by

$$A = \frac{R_f + R_1}{R_1} \times \frac{r_d}{r_d + R_{in}}$$

If V_0 drops below -0.7 V (after removing offset with C_2 and R_2), transistor Q1 open and capacitor C_3 is discharged, $V_G = 0$. In this case, FET F1 opens and its resistance is 0. In that case the overal gain is

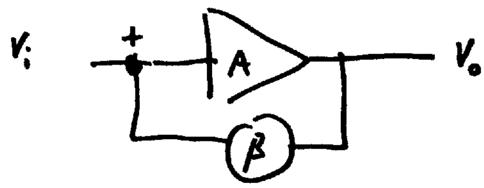
$$A = \frac{R_f + R_1}{R_1} \cdot \frac{0}{0 + R_{in}} = 0$$

If V_0 never drops below -0.7 V (after HPF), transistor Q1 stays closed, and C_3 charges to $-V_{cc}$ (through R_3). In that case V_G reaches $-V_{cc}$ and FET F1 close, $r_d = \infty$, and the overal gain is

$$A = \frac{R_f + R_1}{R_1} \cdot \frac{\infty}{\infty + R_{in}} = \frac{R_f + R_1}{R_1}$$

Waveform signal generators

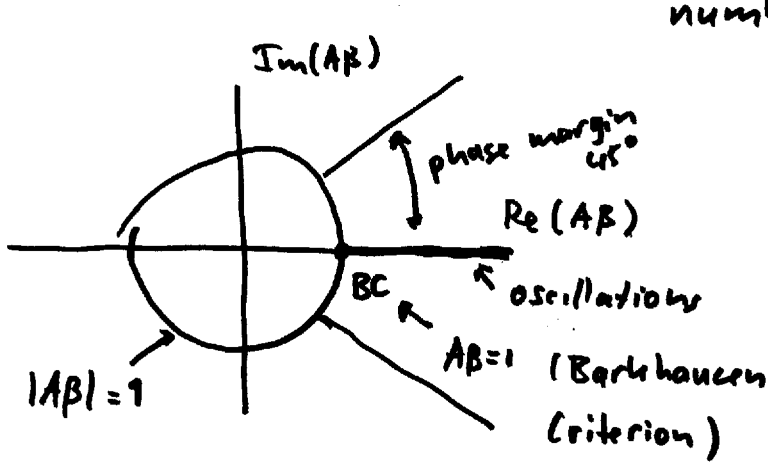
Going back to feedback theory (Electronics II)



The gain of this circuit (positive feedback, see the + sign) is given by

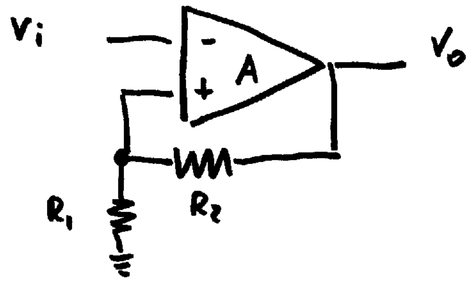
$$\frac{V_o}{V_i} = \frac{A}{1 - A\beta}$$

We found that the circuit will oscillate when $A\beta = 1$. More precicely, we made the Nyquist plot of $A\beta$ (which in general is a complex number)



Everything inside the phase margin, with $|A\beta| > 1$ has <u>risk</u> of oscillations. Everything with $A\beta$ real and > 1 <u>will</u> oscillate ($V_o \neq 0$ for $V_i = 0$)

As an example



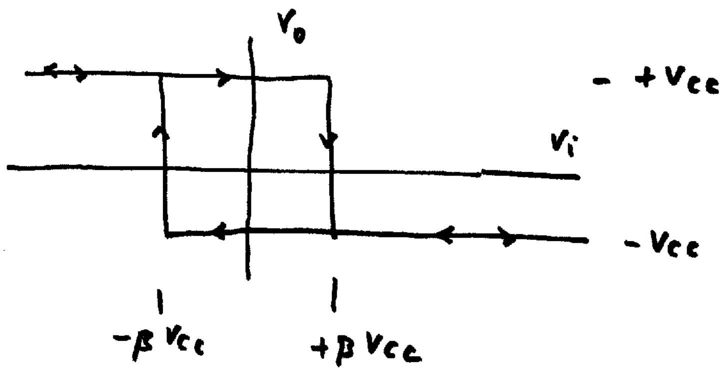
$$\beta = \frac{V_p}{V_o} = \frac{R_1}{R_1 + R_2}$$

↑ real number

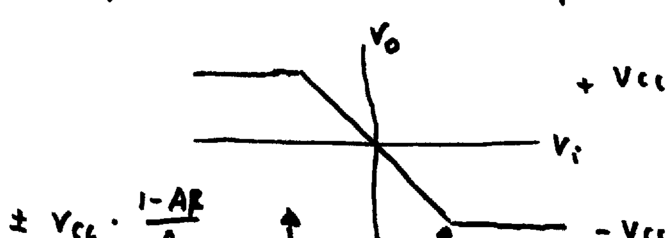
If $A = \infty$, $A\beta$ is always real and > 1. It will thus oscillate, even at 0 Hz (DC). Better to say that it saturates at DC ($A\beta = 1$ means marginally maintaining signal, $A\beta > 1$ means oscillations with increasing amplitude).

In fact, the above circuit can saturate at either value of the power supply voltages. $\pm V_{cc}$.

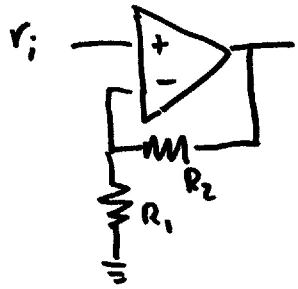
It is a bi-stable circuit, also known as 'memory'.



On the other hand, if $A \neq \infty$, then we have the possibility to have $A\beta < 1$, the circuit is stable:

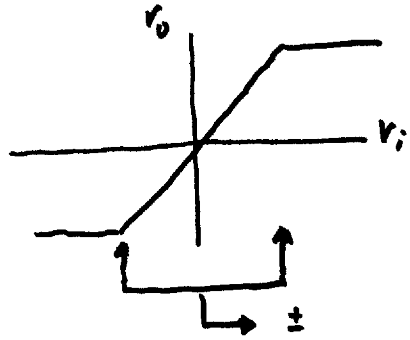


Negative feedback tends to be stable.



$$\beta = -\frac{R_1}{R_1+R_2}$$

$A\beta$ is always < 1. In fact, it is < 0.

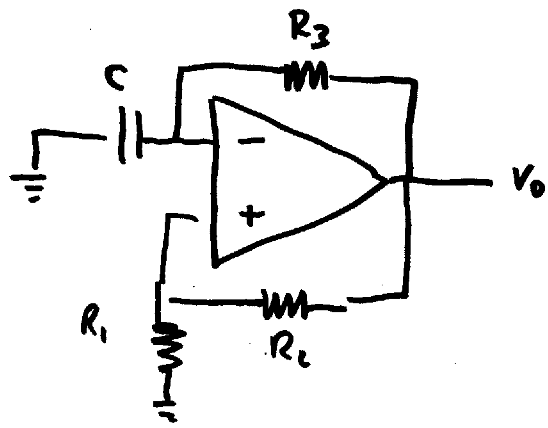


$$\frac{v_o}{v_i} = \frac{A}{1+A\beta}$$

$$\pm \frac{1+A\beta}{A} \cdot V_{cc}$$

With 'mixed feedback' (both positive and negative) we can make a circuit that is only unstable at certain frequencies.

<u>LC - oscillator / relaxation oscillator / astable circuit</u>



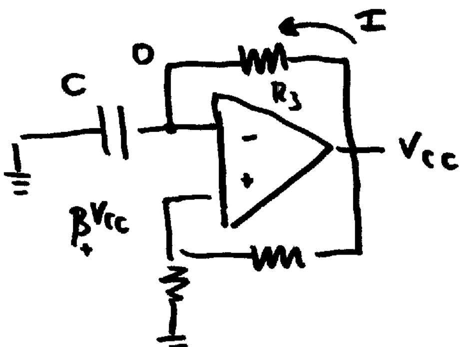
$$\beta = \frac{R_1}{R_1+R_2} - \frac{1/j\omega C}{1/j\omega C + R_3}$$

$$\beta = \underbrace{\frac{R_1}{R_1+R_2}}_{\nearrow \beta_+} - \underbrace{\frac{1}{1+j\omega RC}}_{\nearrow \beta_-}$$

It is obvious that for $\omega = 0$ $\beta < 0$ (negative feedback) and the circuit is stable. It will not saturate at $\pm V_{cc}$! For $\omega = \infty$, the feedback is positive and it will oscillate (if $\frac{R_1}{R_1+R_2} \cdot A > 1$).

Yet, this circuit is easier to analyze in the time domain (instead of the frequency domain given above).

Imagine the capacitor is discharged ($Q = 0$, thus $\Delta V = Q/C = 0 \Rightarrow V_n = 0$), and $V_o = +V_{cc}$. We have



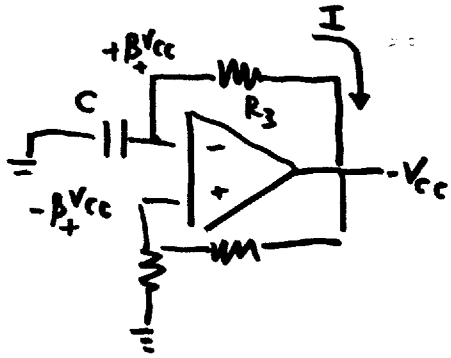
the situation as shown on the left. A current will flow that is (initially) given by $I = (V_{cc} - 0)/R_3$.

$V_p > V_n$, thus $V_o = V_{cc}$

This current will charge capacitor C, $Q = \int I(t)\,dt$.
The voltage at V_n will rise, until it exceeds $V_P = \beta_+ V_{cc}$.
In that case, $V_n > V_p$ and the output will commutate to $-V_{cc}$. This commutation will do nothing to the charge contained within C, which thus

keeps a charge such that $V_n = 0 + \frac{Q}{C} = \frac{\beta_+}{V_{cc}}$.

We now have the following situation

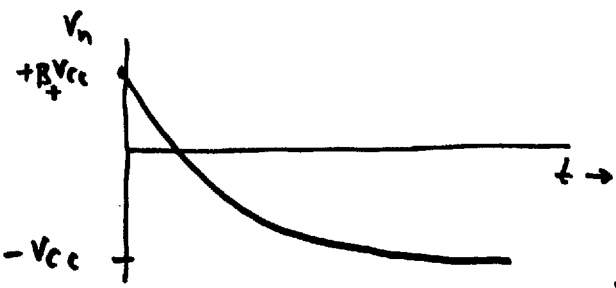


A current will flow from V_n to V_o through R_3. This current <u>must</u> come from C, which will thus be discharged. When C

$V_n > V_p$, thus $V_o = -V_{cc}$

discharges, $\Delta V = Q/C$ will drop and the current will be less (because ΔV of R_3 decreases). We recognize an exponential decay behavior.

- $V_n(t=0) = \beta_+ V_{cc}$
- $\tau = R_3 C$
- $V_n(t=\infty) = -V_{cc}$ (then $I = \frac{V_n - V_o}{R_3} = 0$)



$$V_n = -V_{cc} + (1+\beta_+) V_{cc} \exp(-t/\tau)$$

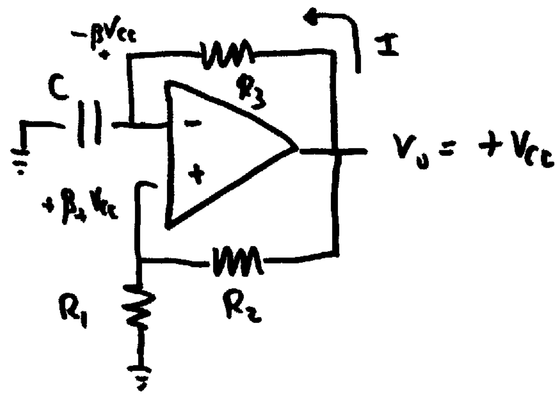
However, this never reaches $-V_{cc}$, because, when V_n drops below V_p ($= -\beta_+ V_{cc}$), the out will switch from $-V_{cc}$ to $+V_{cc}$. When does this occur?

$$V_n = -V_{cc} + (1+\beta_+)V_{cc}\ \exp(-t/\tau) = -\beta_+ V_{cc}$$

$$\Longrightarrow$$

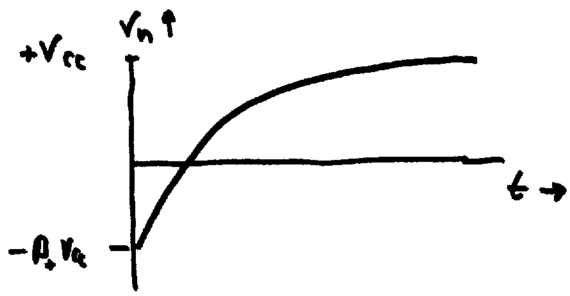
$$t = \tau \ln\left(\frac{1+\beta_+}{1-\beta_+}\right) = R_3 C \ln\left(\frac{1+\beta_+}{1-\beta_+}\right)$$

At this point we have :



A current flows into C and charges it. Exponential behavior

- $V_n\ (t=0) = -\beta_+ V_{cc}$
- $\tau = R_3 C$
- $V_n\ (t=\infty) = +V_{cc}$ (then $I = \frac{V_n - V_o}{R_3} = 0$)



$$V_n = V_{cc} - (1+\beta_+)\exp(-t/\tau)$$

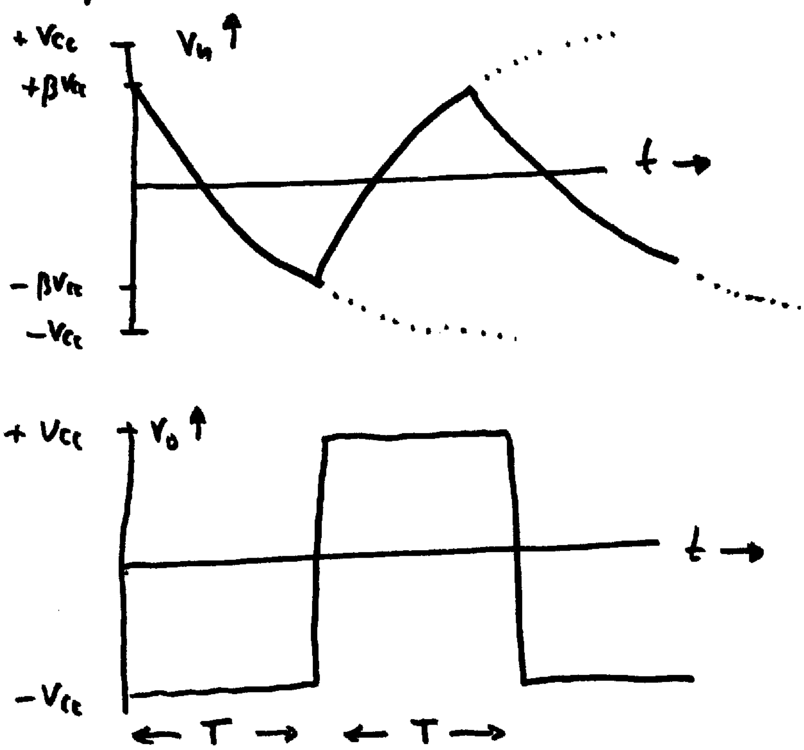
when this reaches V_p ($= +\beta_+ V_{cc}$) the circuit will swith again :

$$V_n = V_{cc} - (1+\beta_+)\exp(-t/\tau) = +\beta_+ V_{cc}$$

$$\Longrightarrow$$

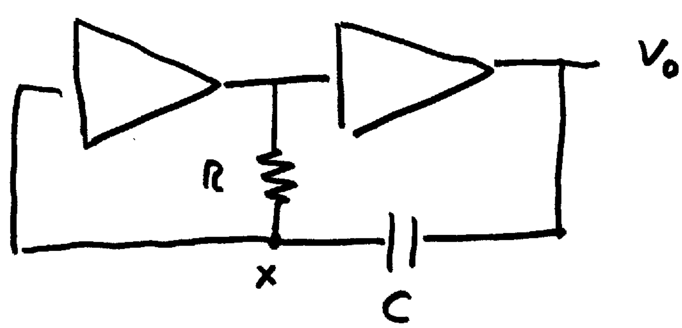
$$t = \tau \ln\left(\frac{1+\beta_+}{1-\beta_+}\right) = R_3 C \ln\left(\frac{1+\beta_+}{1-\beta_+}\right)$$

In summary:

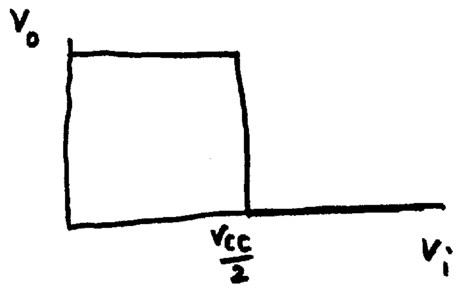


$$f = \frac{1}{2T} = \frac{1}{2R_3C \ln\left(\frac{1+\beta_+}{1-\beta_+}\right)}$$

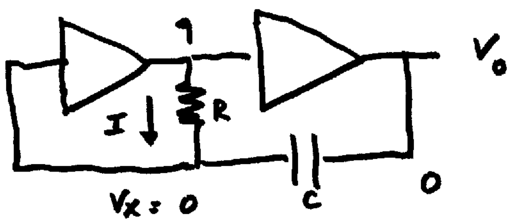
Logic - gate oscillator



Each opamp has the negative gain like shown below



Imagine C is 'empty', $\Delta V = 0$. Imagine $V_o = 0$
We have the following situation (voltages in unit of V_{cc})

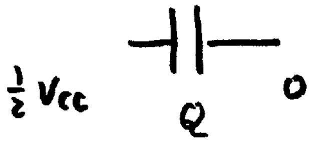


The resistance R has $+V_{cc}$ on one side and 0 on the other. Ohm's Law tells us that a current $I = V_{cc}/R$ will flow. This current cannot enter the first opamp and is thus used to charge C. Charging it will cause ΔV of C to rise, C has one foot at $V_o = 0$, thus the other foot, V_x, must rise. This reduces the voltage drop of R and reduces the current. We recognize a classical relaxation system.

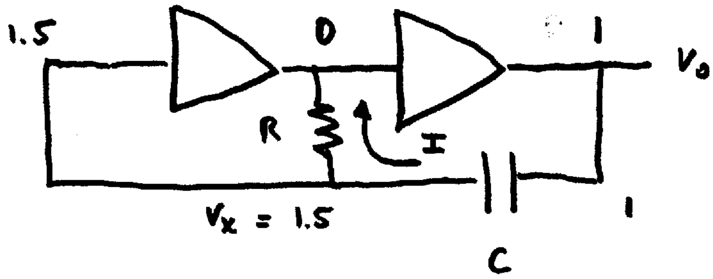
- $V_x(t=0) = 0$, $V_x(t=\infty) = V_{cc}$, $\tau = RC$

When $V_x = \frac{1}{2} V_{cc}$, the first opamp will switch

Just before that, the charge in C is such that



$\Delta V = \frac{1}{2} V_{cc}$. This charge cannot dissipate instantaneously, so we have the situation directly after switching equal to



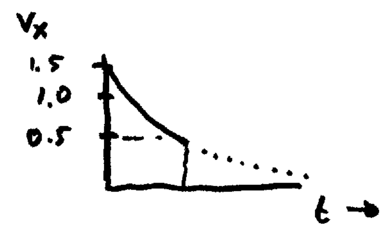
A current will flow that discharges the capacitor C. Exponential behavior

$t = 0$: $V_x = 1.5\ V_{cc}$

$t = \infty$: $V_x = 0$ $(I = 0)$

$\tau = RC$

thus



$$V_x = 1.5\ V_{cc} \exp(-t/RC)$$

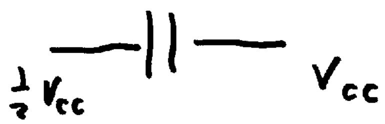
This continues until V_x drops below $0.5\ V_{cc}$. (which will make opamp 1 switch). When will this happen?

$$1.5\ V_{cc} \exp(-t/RC) = 0.5\ V_{cc}$$

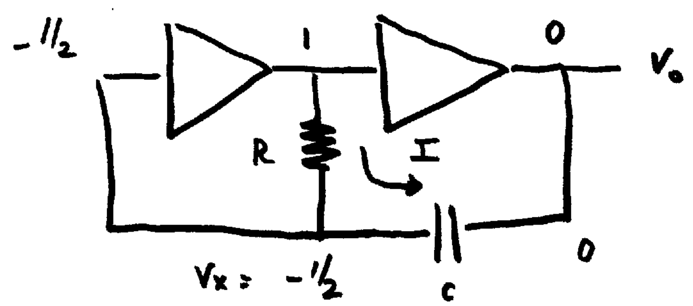
thus

$$t = RC \ln(3)$$

Just before switching, the capacitor feels a drop like this.



This drop will be maintained upon switching. We have the following situation



Exponential behavior charging current I.

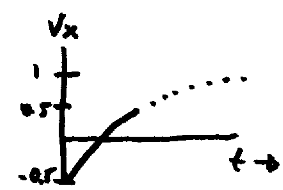
$$t = 0 : \quad V_x = -0.5\, V_{cc}$$

$$t = \infty : \quad V_x = V_{cc} \quad (I = 0)$$

$$\tau = RC$$

thus

$$V_x = V_{cc} - 1.5\, V_{cc} \exp(-t/RC)$$



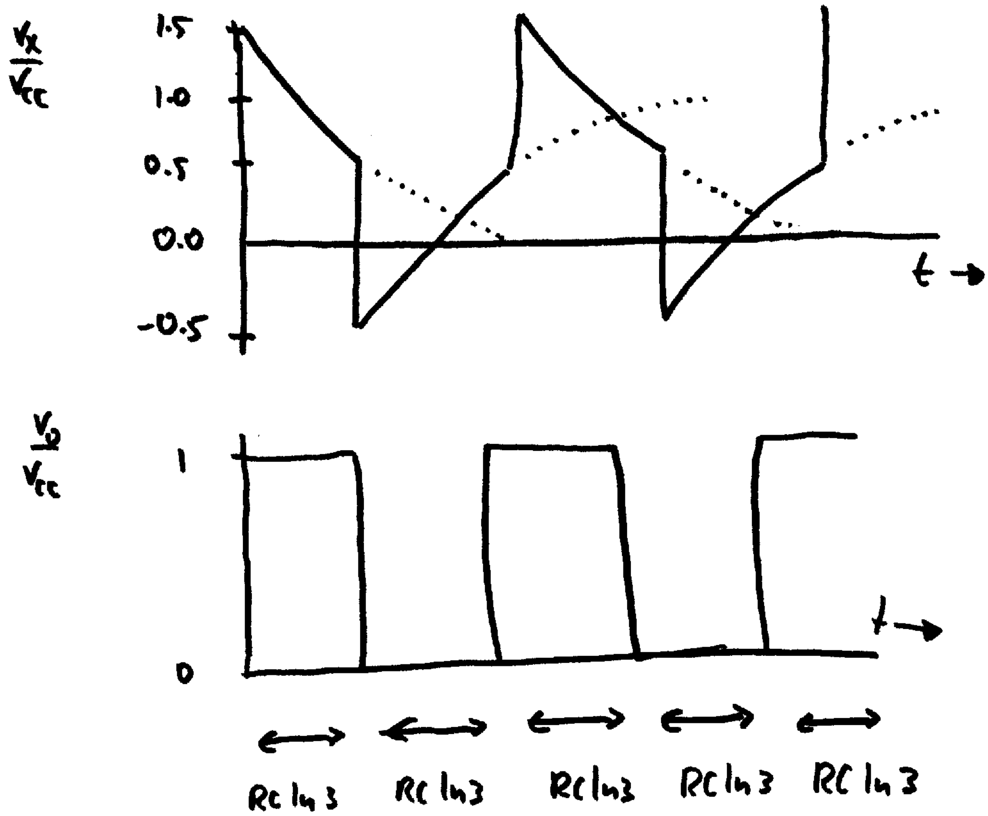
V_x will rise until $0.5\, V_{cc}$, a point at which a switch will occur again

$$V_{cc} - 1.5\, V_{cc} \exp(-t/RC) = 0.5\, V_{cc}$$

thus

$$t = RC \ln(3)$$

In summary



NE555 timer circuits.

One of the circuits of the NE555 family is an oscillator. (See Instrumentation lectures)

Quartz crystal oscillator

These are used as frequency references in computers, etc. They are made of quartz (SiO_2) because of their physical properties

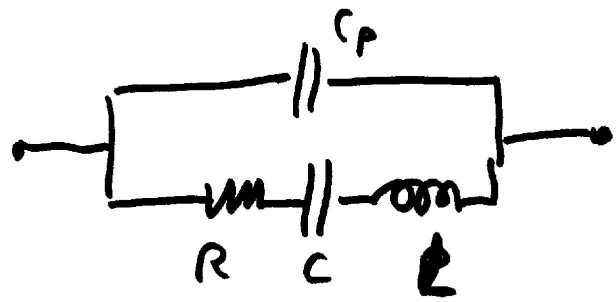
- crystal
- non inversion symetric $\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} \right.$ gives a different crystal $\left. \right)$

Because of this combination, it has piëzo-electric effect. That is, it couples mechanical deformations to electrical signals and vice versa.

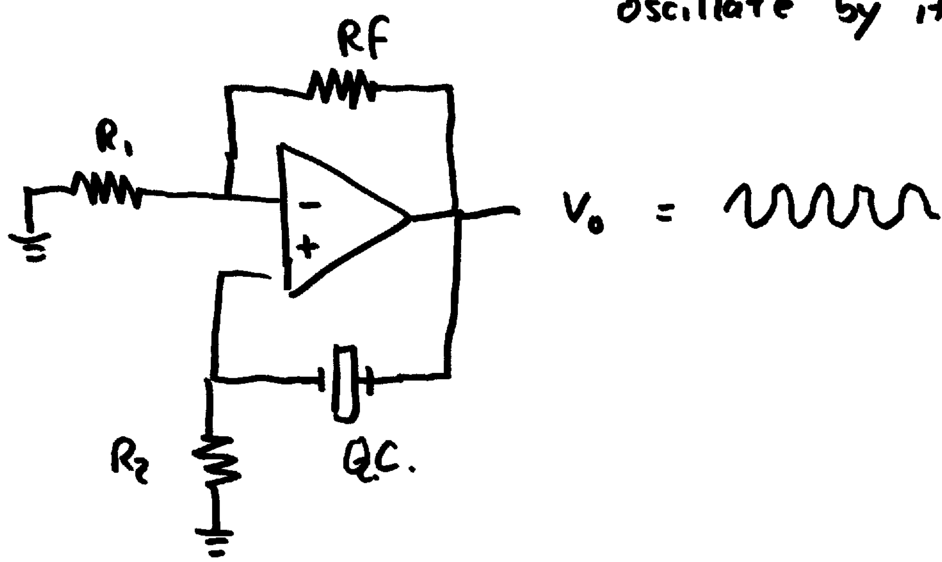
So, if we apply an electrical sine, the crystal will deform with this sine wave. Moreover, any crystal has a natural ('resonance') frequency, depending on the thickness of the crystal. Thus, crystals exist for 10 MHz (thick), 50 MHz (thinner), etc.

This mechanical behavior can be modelled by an equivalent electrical circuit, called Butterworth van Dyke (BVD):

Q.C. ≡



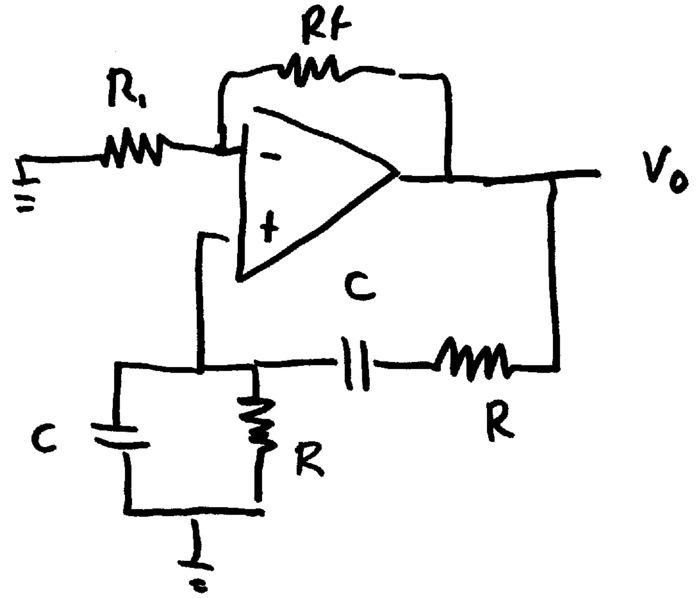
when we put this in a feedback circuit it can start to oscillate. (It does <u>not</u> oscillate by itself!)



See exercise. Assume $C_p = \emptyset$ and $R = \emptyset$. Show that the circuit oscillates at

$$\omega = \frac{1}{\sqrt{LC}}$$

Wien oscillator



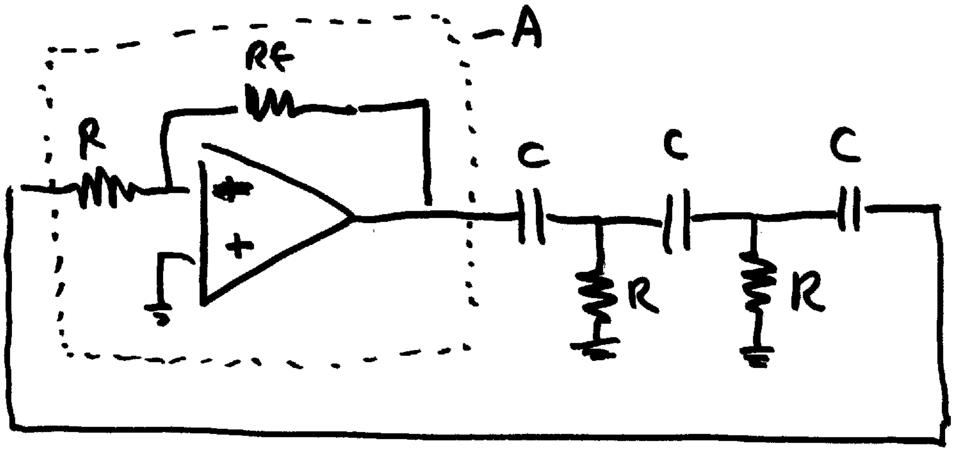
Barkhausen criterion $A\beta(\omega) = 1$ (El. II)

For this frequency the circuit will oscillate

For a good choice of Rf (see exercise), this can occur.

$$R_f = 2R_1 \quad , \quad \omega = \frac{1}{RC}$$

Phase - shift oscillator



This circuit will oscillate when Barkhausen Criterion is met, namely at

$$\omega = \frac{1}{\sqrt{6}\,RC}, \quad R_f = 29\,R$$

Other oscillators:

- Hartley
- Colpitts.

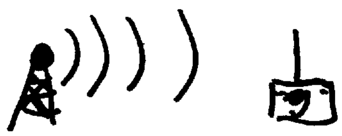
They always work on the same principle

$\boxed{\text{Barkhausen Criterion.}}$

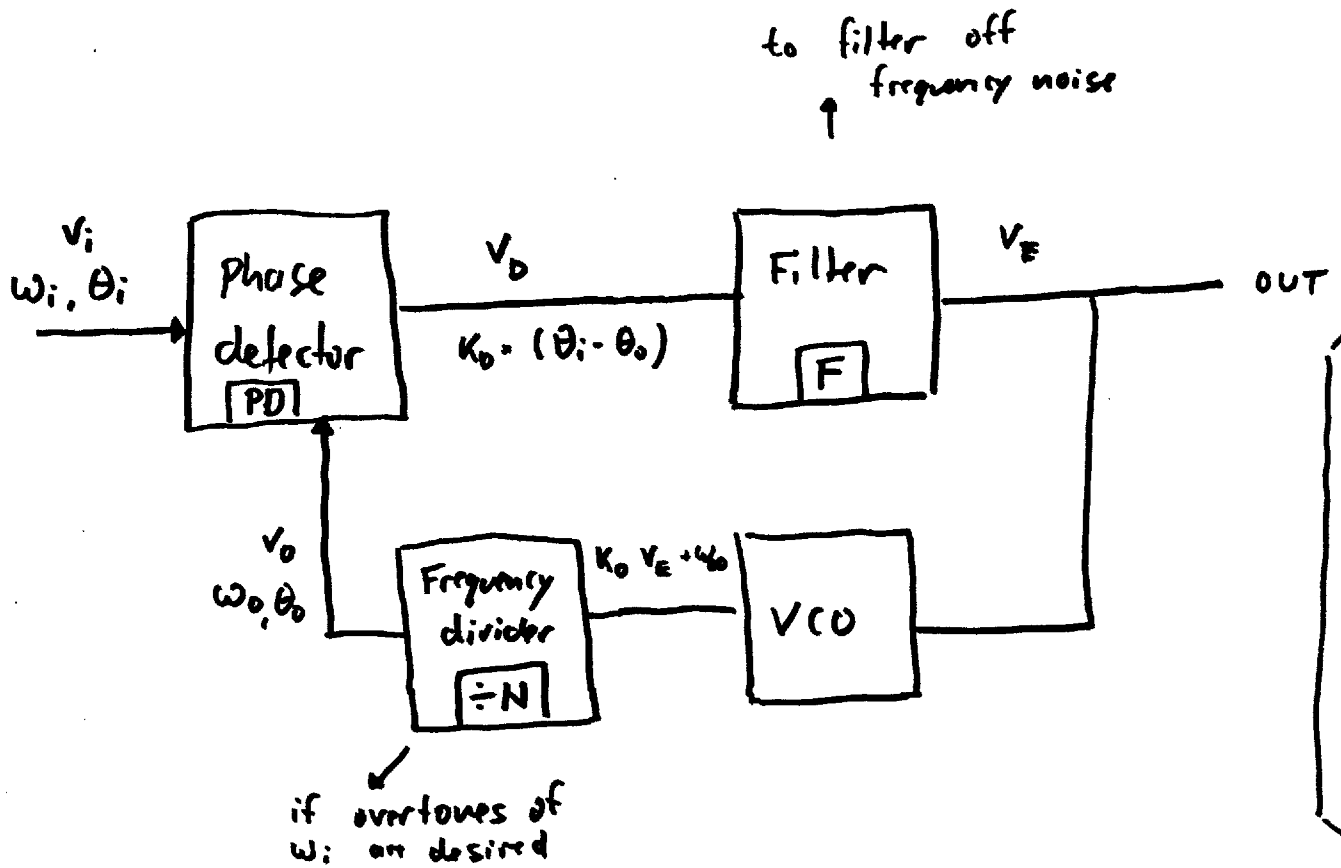
Phase-locked loops

We already know feedback systems where the signal is voltage. A PLL is a system where the feedback signal is frequency. To be more precise, the system maintains the phase constant, and this causes the signal to catch (lock into) the input signal frequency.

The most obvious application of this is to FM radio signals. The receiver



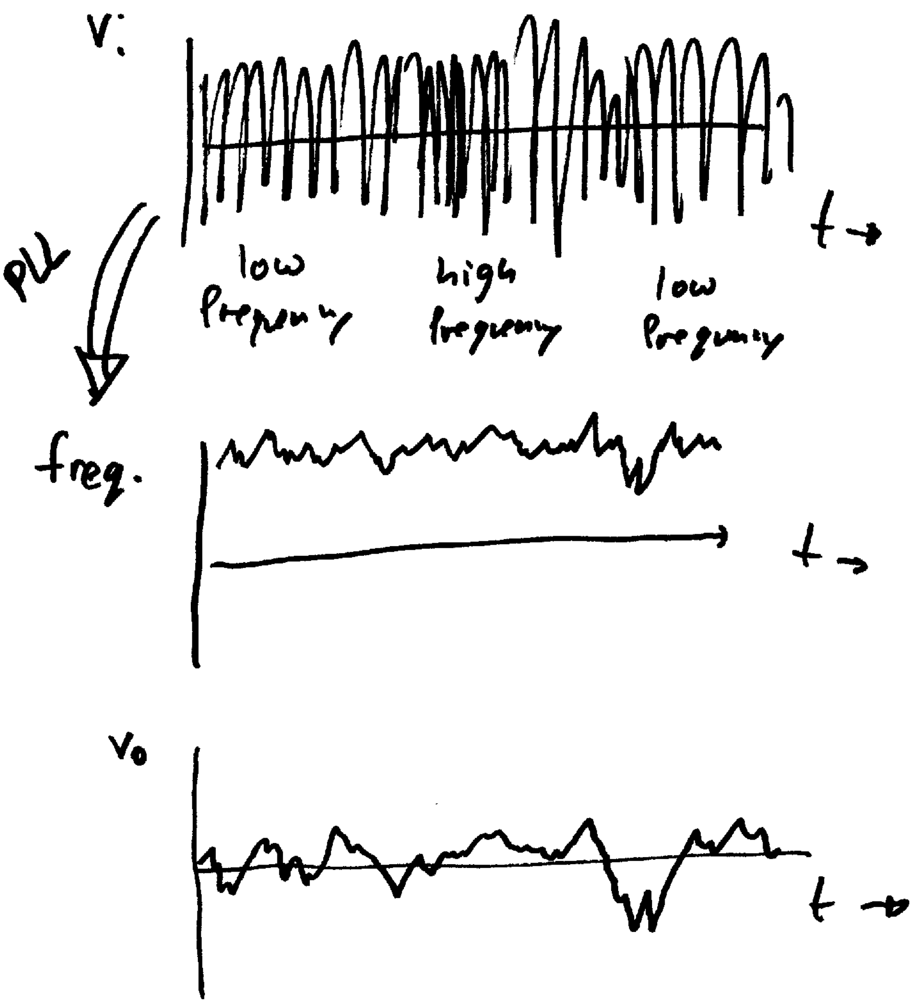
demodulates the signal; deviations from the carrier frequency are translated into audio acoustic signals



FM radio example: v_E is proportional to frequency deviation in FM signals.
(v_E is the audio signal used to modulate the frequency)

BASIC ingredients of PLL

"Should ω_i change, the phase shift between v_o and v_i will start to increase, changing v_D and, hence, the control voltage v_E. This change is designed to adjust the VCO until ω_o is brought back to the same value as ω_i. This self-adjusting ability by the feedback loop allows ~~the~~ the PLL, once locked, to track input frequency changes." (Sergio Franco, "design with operational amplifiers and analog integrated circuits")



Signal amplitude is irrelevant!

frequency of incoming signal

output of PLL signal sent to audio power amplifier

In this chapter we will learn how this is achieved.

The basic components are

* a phase detector. Converts frequency into a voltage that is proportional to the phase. $V_o = \alpha\,\theta_i$. $[\,V_i = \sin(\theta_i),\ \theta_i = \omega t\,]$
  In case feedback is used, the detector receives two frequencies

$V_i = \sin(\theta_i)$, $\theta_i = \omega_i t$

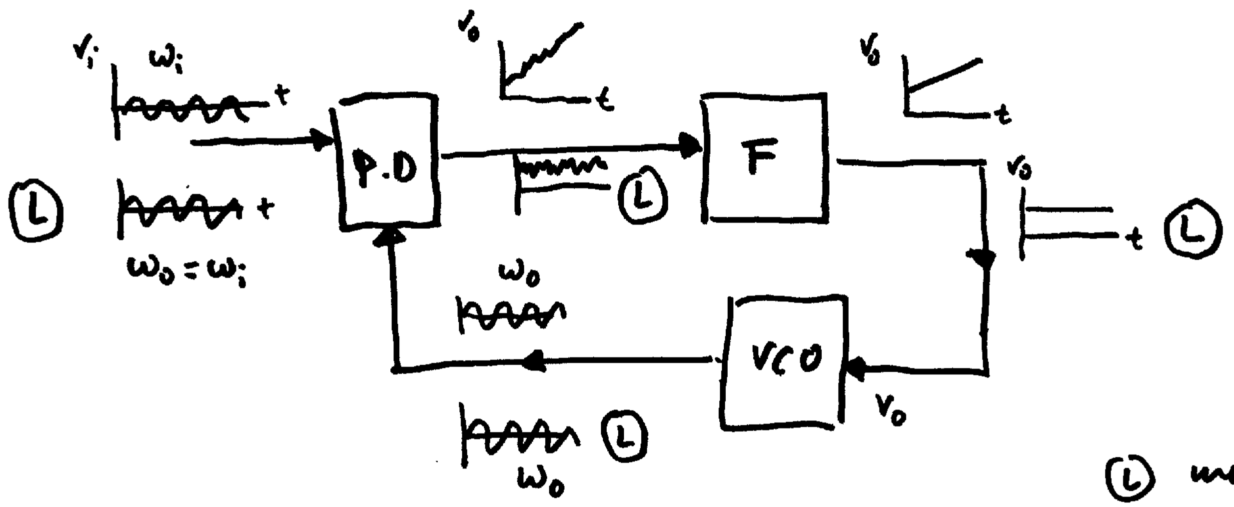
$V_o = \sin(\theta_o)$, $\theta_o = \omega_o t + \theta_{oo}$

and the output of the P.D. is

$$V_o = \alpha(\theta_i - \theta_o)$$

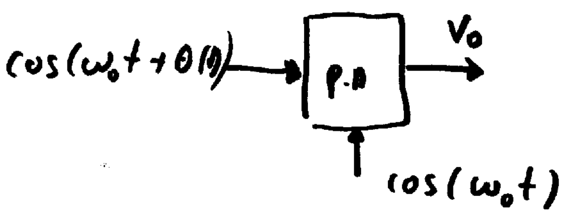
* A filter, F. In most cases, the output of the P.D is passed through a filter

* A voltage controlled oscillator, VCO. This can be seen as the opposite of the P.D. A voltage is converted into a frequency.



L means: when signal locked

Examples of P.D.

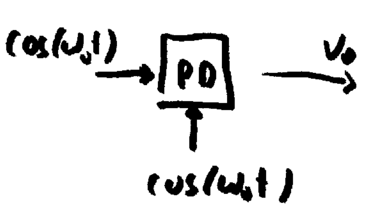


input 1: $\cos(\omega_o t + \theta(t))$

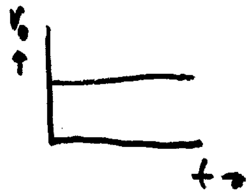
input 2: $\cos(\omega_o t)$

EXAMPLES OF P.D. SIGNALS

i) $\theta(t) = 0$

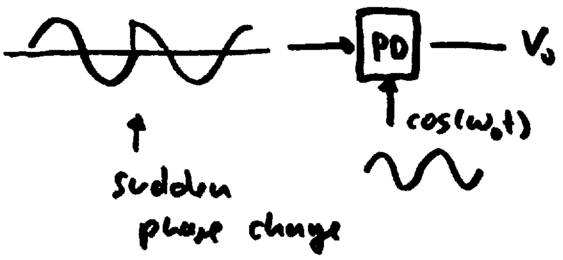


$V_0 = \theta$

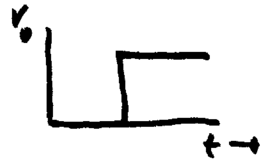


ii) $\theta(t) = u(t)$

sudden phase change of input signal

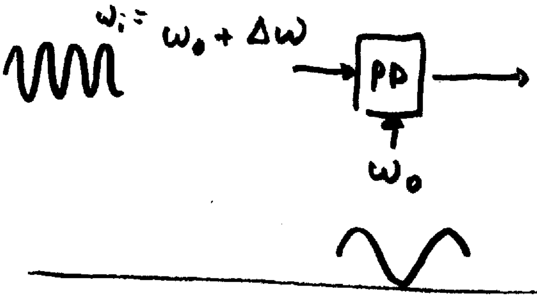


$V_0 = u(t)$

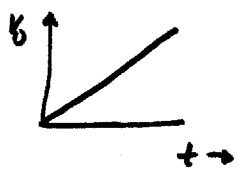


iii) $\theta(t) = \Delta\omega\, t\, u(t)$

sudden change in frequency of input signal

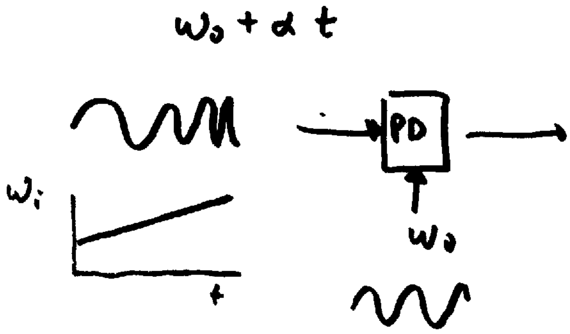


$V_0 = \Delta\omega\, t$

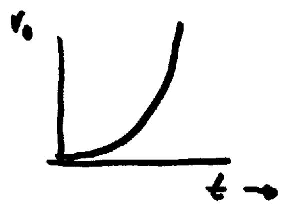


iv) $\theta(t) = \alpha t^2$

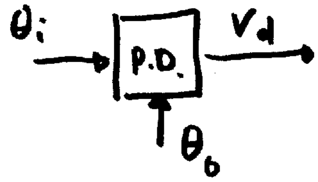
ramping frequency



$V_0 = \alpha t^2$

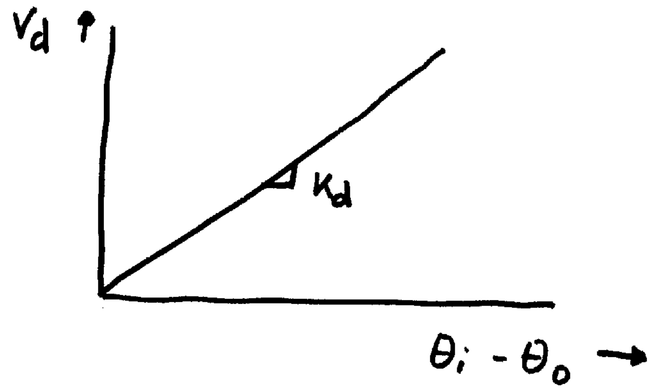


The parameter characterizing a P.D is called K_d, the sensitivity of P.D.

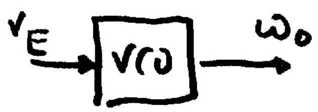


$$V_d = K_d \times (\theta_i - \theta_o)$$

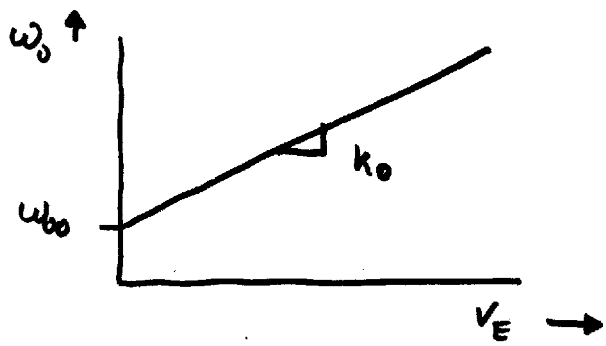
The unit of K_d is V/rad



The parameter characterizing a voltage-controlled oscillator (VCO) is K_o, sensitivity of VCO



$$\omega_o = K_o V_E + \omega_{oo}$$



The unit of K_o = $\frac{rad}{s}$ / V

The parameters of the filter are like any other filter. Normally they are Low-pass filters, so as to filter frequency noise and only let through low frequency signal.

The frequency divider is optional.

To analyze a PLL we best use the LAPLACE TRANSFORM of the signals in the loop. If we realize that the phase is the integral of frequency,

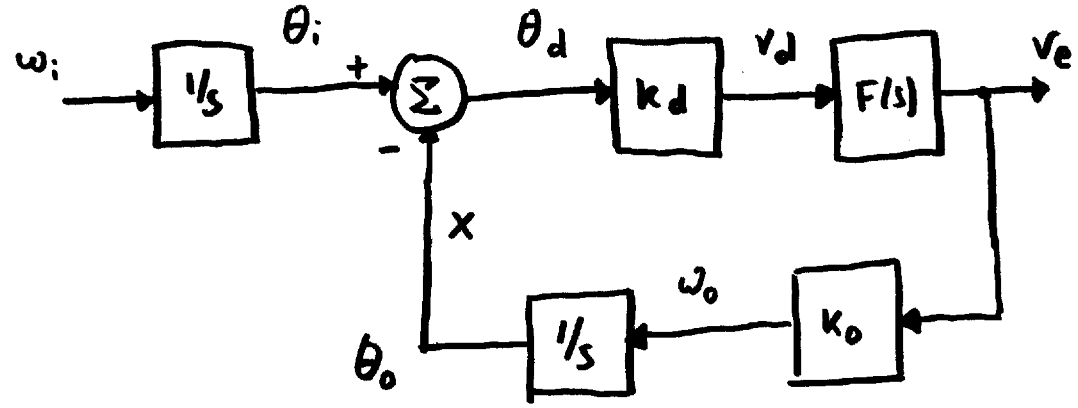
$$\boxed{\theta(t) = \theta(0) + \int_0^t \omega(\tau)\,d\tau}$$

and that the Laplace transform of the integral of a function is the Laplace transform of the function itself, divided by s:

$$\boxed{\mathcal{L}\left(\int f(t)\,dt\right) = \frac{1}{s}\,\mathcal{L}(f(t))}$$

it is easy to write the blocks of a PLL in Laplace transform:

when 'Locked' :



If we open the loop at point x, we find a loop gain $T(s)$,

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{k_d\, F(s)\, K_o}{s} \equiv T(s)$$

$$= K_v \frac{F(s)}{s}$$

$$K_v = K_d\, K_o$$

Closing the loop, we find

$$\theta_o(s) = (\theta_i(s) - \theta_o(s)) \cdot K_d \cdot F(s) \cdot K_o \cdot \frac{1}{s}$$

or

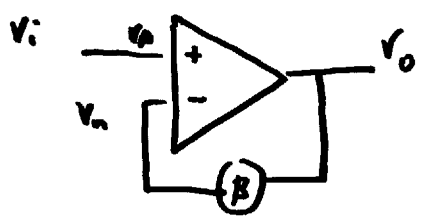
$$H(s) \equiv \frac{\theta_o(s)}{\theta_i(s)} = \frac{K_v\, F(s)}{s + K_v\, F(s)} = \frac{T(s)}{1 + T(s)}$$

Another interesting function is $v_e(s)/\omega_i(s)$. Given the fact that $\theta_i = \omega_i/s$, and $\theta_o = v_e K_o/s$, we find

$$\frac{v_e(s)}{\omega_i(s)} = \frac{1}{K_o} H(s)$$

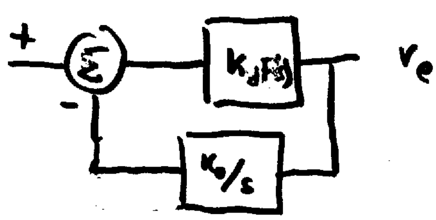
which allows us to find the voltage change $v_e(s)$ ("the audio signal") in response to an input frequency change $\omega_i(s)$, as in FM demodulation.

OBS: The figure of page 7 shows that a PLL is a <u>negative</u> feedback system, with $x_i = \theta_i$ and $x_o = \theta_o$. The PLL will force θ_o to track θ_i just as an op amp forces v_p to track V_n. We can see a PLL as a phase-follower, just as we considered an op-amp a voltage-follower. The fact that it also forces ω_o to follow ω_i is a consequence of this phase-follower action, along with the phase-frequency relationship $\omega = d\theta/dt$

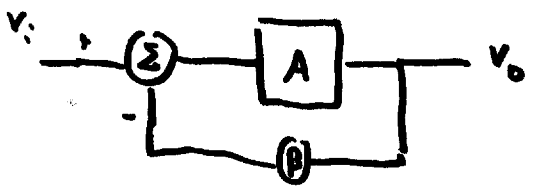


opamp

$\approx$



PLL



PLL without filter $F(s)$ (First-order loop):

$$T(s) = \frac{1}{s/K_v} \qquad \left(= \frac{K_d K_o}{s} \right)$$

$$H(s) = \frac{T(s)}{1+T(s)} = \frac{K_v}{s+K_v} = \frac{1}{1+s/K_v}$$

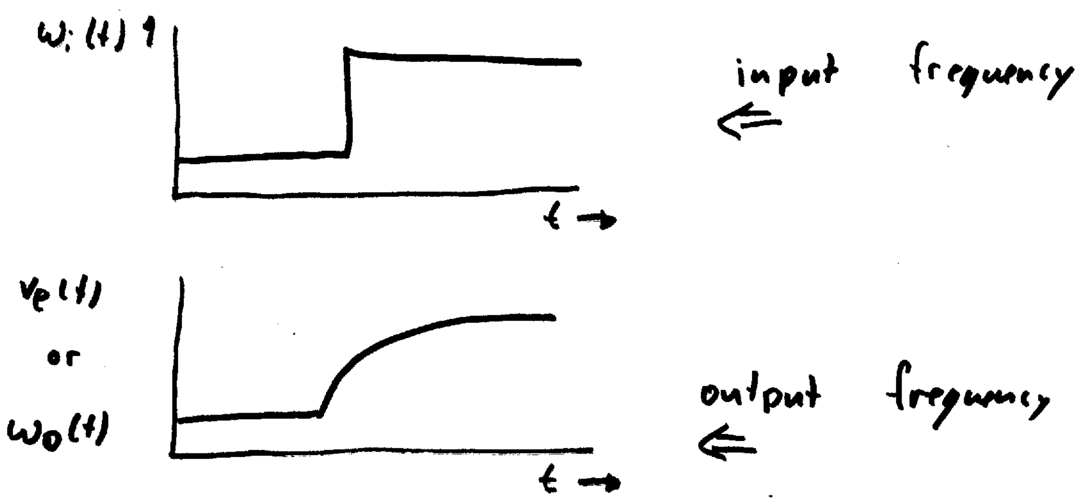
$$\frac{V_e(s)}{\omega_i(s)} = \frac{1}{K_o} H(s) = \frac{1/K_o}{1+s/K_v} = \frac{1/K_o}{1+j\omega/K_v}$$

This is very similar to a filter with cross-over frequency $\omega_x = K_v$. LPF, with DC gain equal to $1/K_o$.

Consequence:

If $\omega_i(t)$ is a step change 

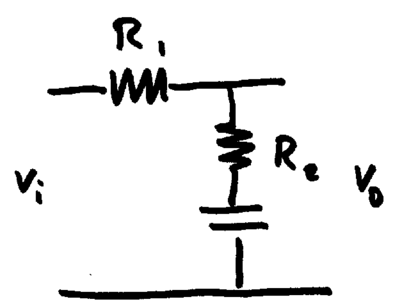
, $V_e(t)$ will be an exponential transient governed by the time constant $\tau = 1/K_v$



PLL with filter (Second-order loop)

Introducing a filter $F(s)$ in the loop can reduce phase noise. However, care has to be taken to avoid instability:

A PLL without $F(s)$ results in a LPF behavior of the system. As will be shown, a PLL with a simple LPF for $F(s)$ will result in a double-pole, second order behavior. As we know from Electronics II, double-pole feedback systems can be unstable. Therefore, filters for PLLs are normally of the lag-lead type:

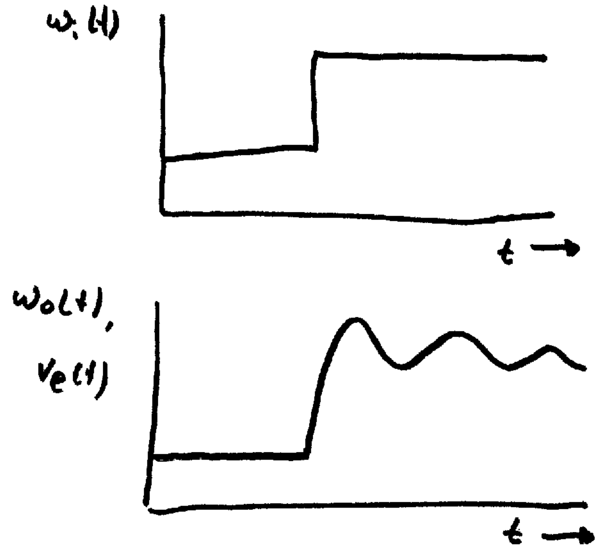


$$F(s) = \frac{1 + s/\omega_z}{1 + s/\omega_p}$$

$$\omega_z = \frac{1}{R_2 C}, \quad \omega_p = \frac{1}{(R_1+R_2)\,C}$$

$$H(s) = \frac{K_v F(s)}{s + K_v F(s)}, \quad \frac{V_e(s)}{\omega_i(s)} = \frac{H(s)}{K_o} = \frac{(K_v/K_o)\cdot(1 + s/\omega_z)}{K_v + (1 + K_v/\omega_z)s + s^2/\omega_p}$$

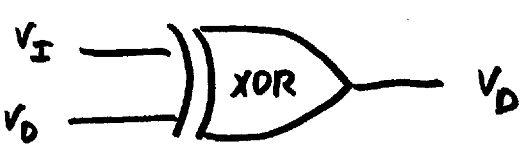
which is quite complicated, but can be made stable. It is the behavior of a damped oscillator, so we can expect a damped-oscillation response



We leave it as a mathematical exercise to determine the exact behavior, for instance the frequency of oscillation and the damping speed.

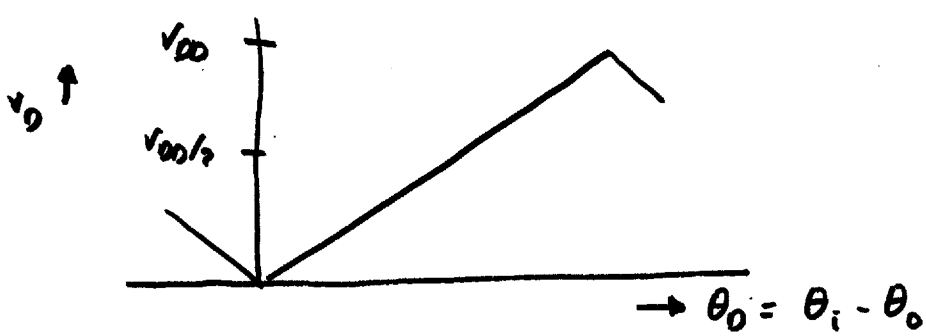
PLL components

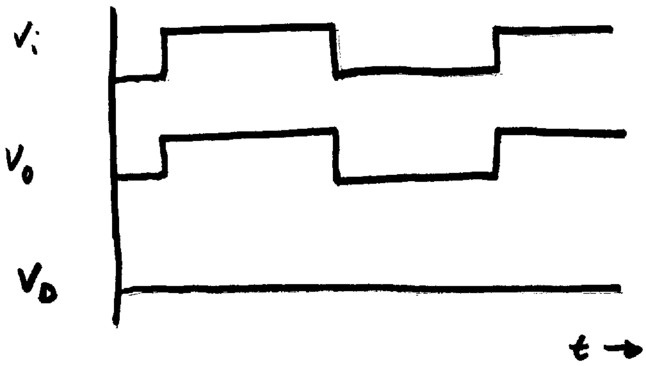
Type I phace detector



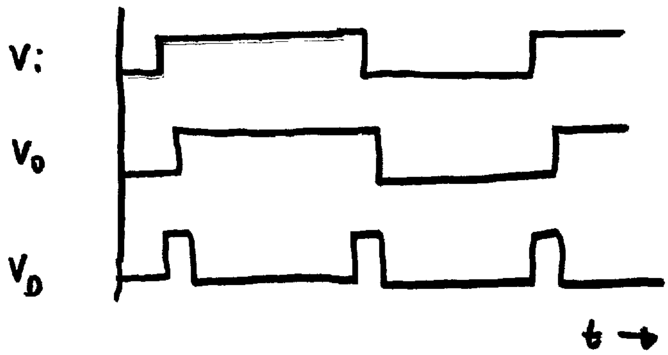
This is an exclusive OR digital circuit.

Result:

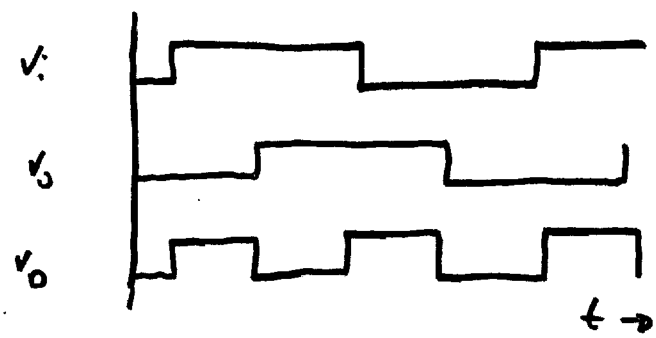




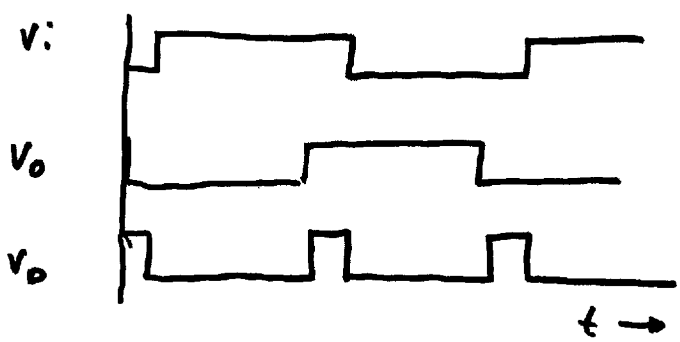
$\theta_o - \theta_i = 0$



$\theta_o - \theta_i \approx \pi/6$

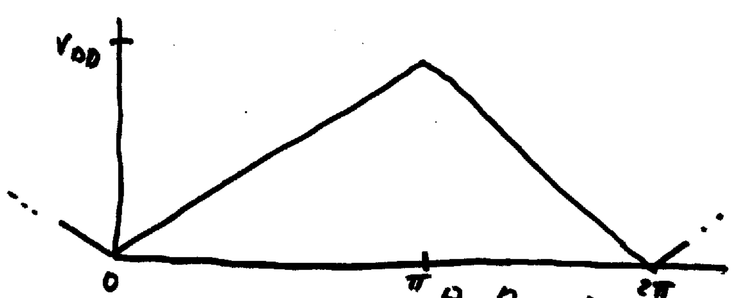


$\theta_o - \theta_i = \pi/2$



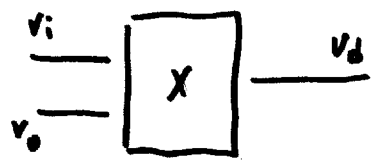
$\theta_o - \theta_i = 5\pi/6$

V_D after LPF :



For this P.D. the waveforms have to be symmetric, with 50% duty cycle.

Analog multiplier P.D.



$$v_i = V_I \cos(\omega t)$$

$$v_o = V_O \cos(\omega t + \phi_d)$$

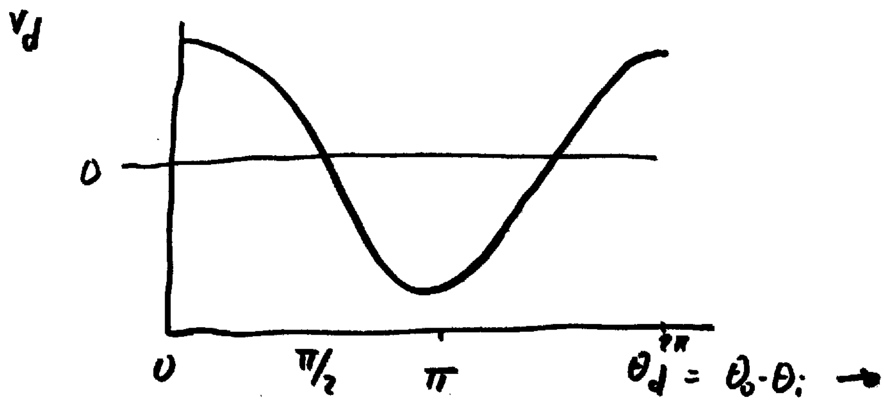
$$V_d = \frac{K \cdot V_I \cdot V_O}{2} \cos(\phi_d) - \frac{K V_I V_O}{2} (2\omega t + \phi)$$

$$\Downarrow$$

LPF

$$\Downarrow$$

$$= K \frac{V_I \cdot V_O}{2} \cos(\phi_d)$$

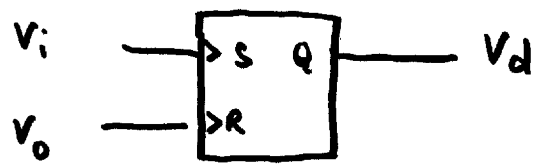


This P.D. is not linear, which makes the system difficult to analyze

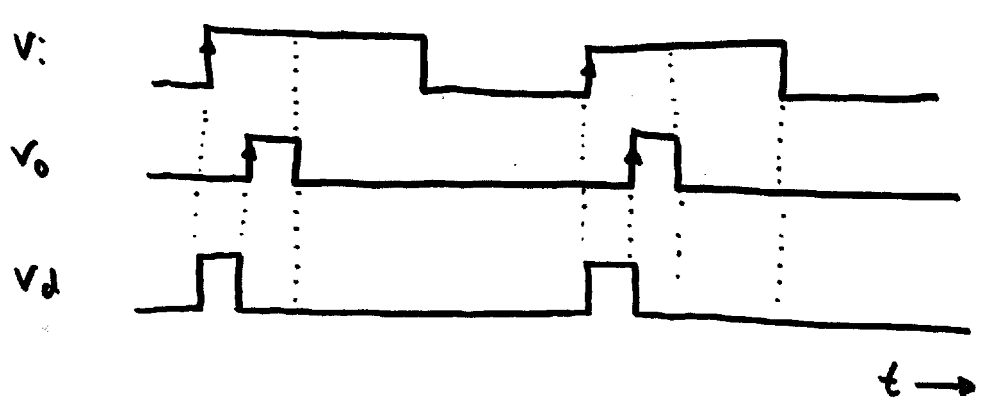
Edge-triggered JK flip-flop P.D.

Type III P.D.

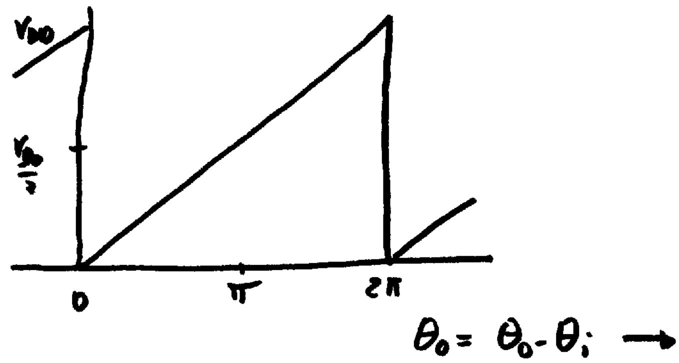
Remember, an edge-triggered S-R flip flop (JK flip flop) responds to <u>changes</u> of input signals, setting Q to 1 when the S (J) input goes from low (0) to high (1), and setting output Q to ∅ when the R (K) input goes from low to high.



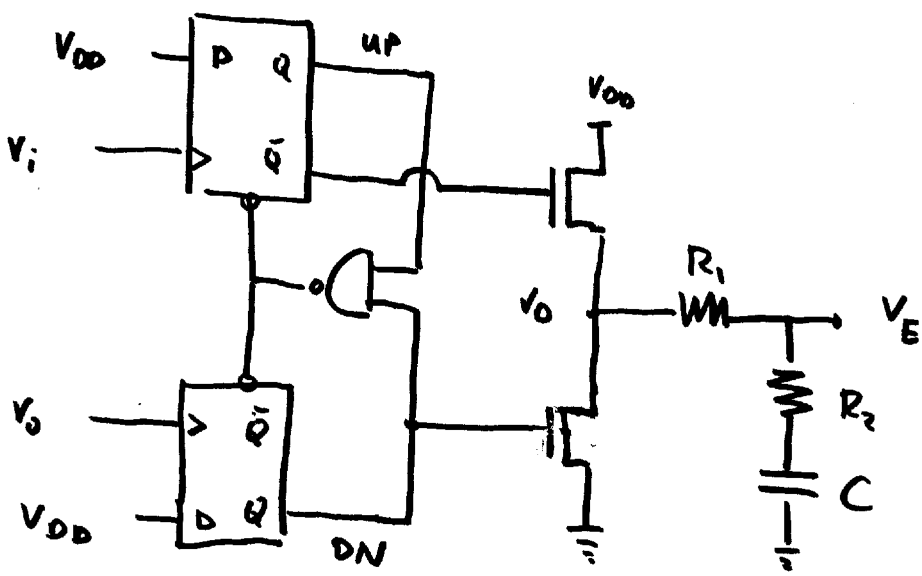
The result is that the output V_d is high from the moment V_i goes high, to the moment V_o goes high. The duty cycles of the waves of V_i and V_o are not important. Example:



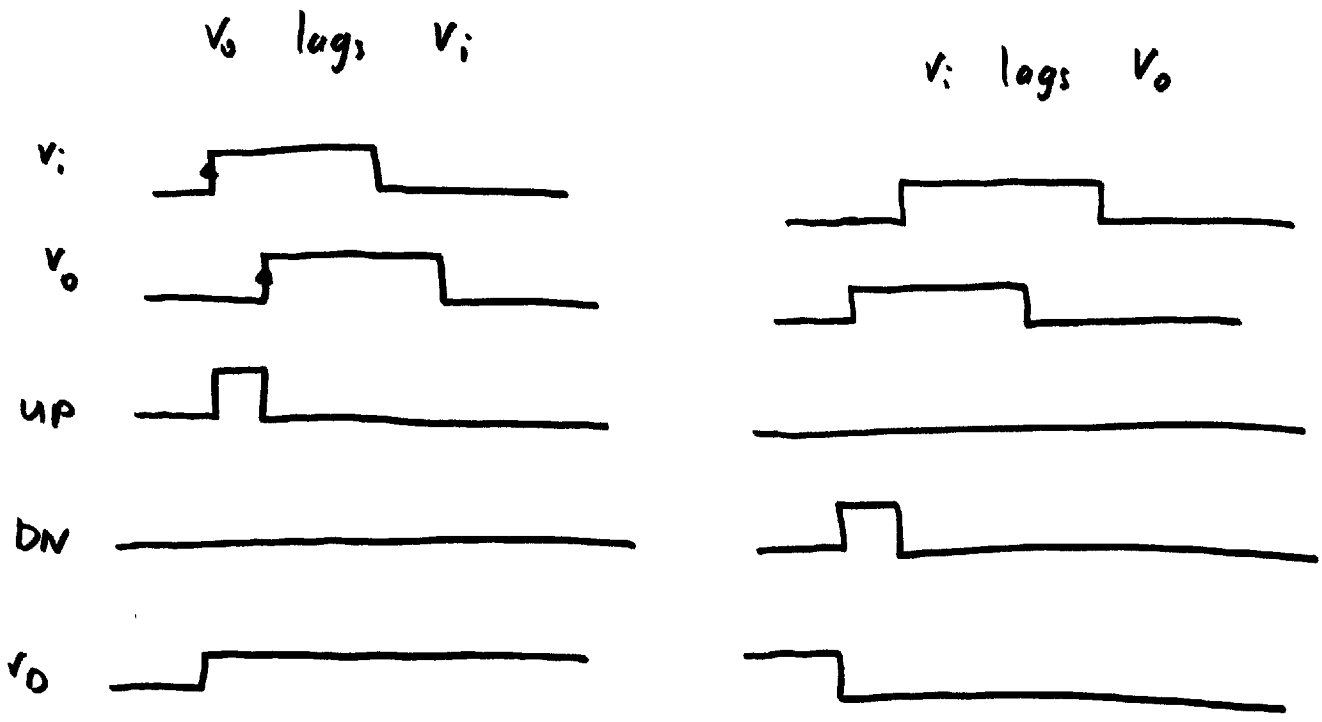
The output V_D (after LPF) of a type III P.D is then



Type II P.D.

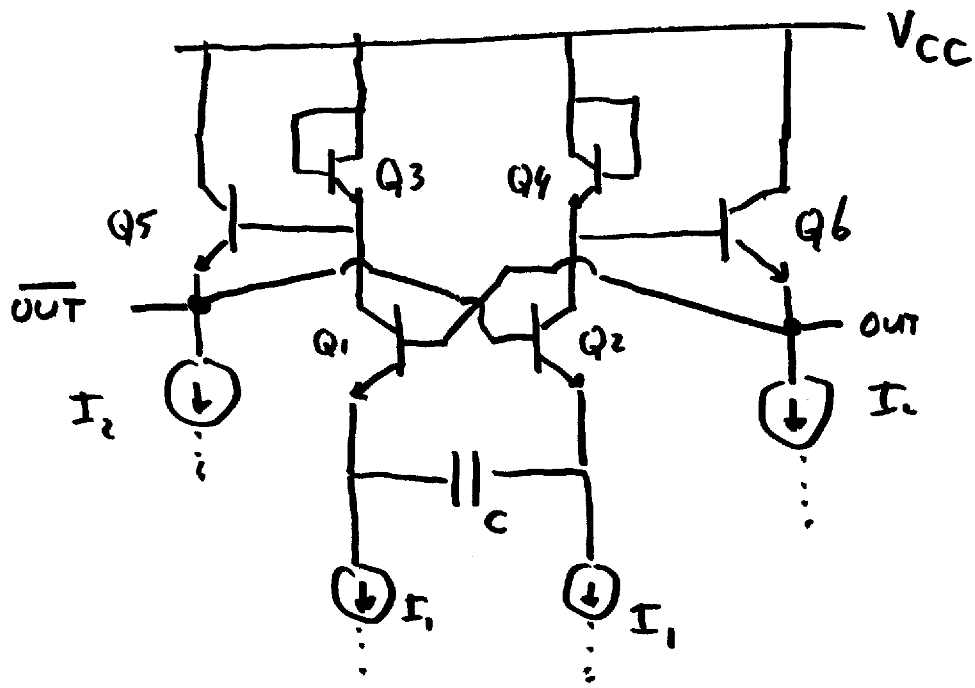


V_i causes an UP signal, increasing V_D voltage. V_o causes a DOWN signal pulse, decreasing V_D.



This make sure not only the frequency $\omega_o = \omega_i$ are equal, like a normal PLL, but also $\theta_o = \theta_i$.

Voltage controlled oscillator example



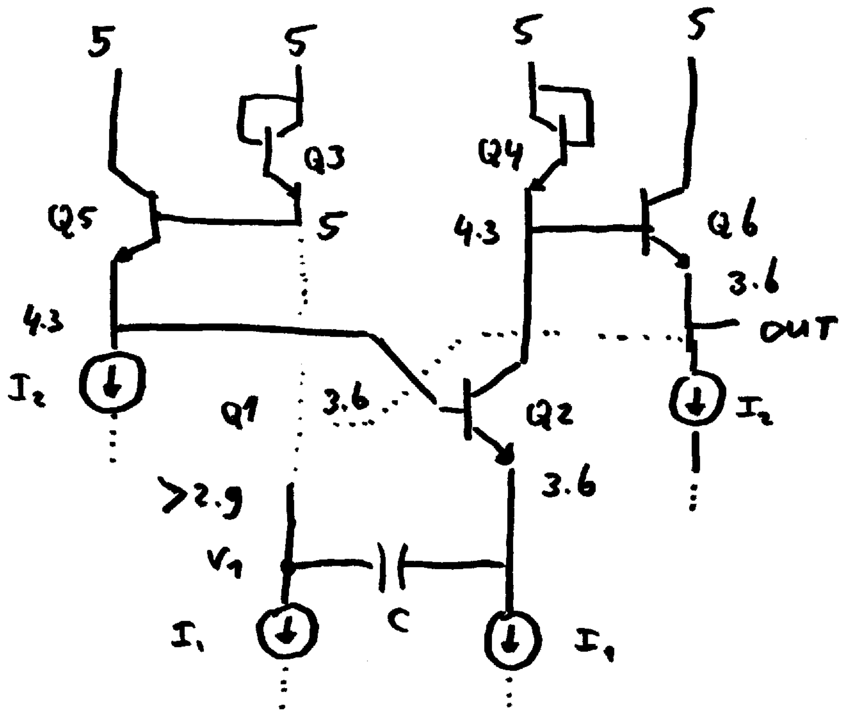
Notes:

1. Q_3 and Q_4 are configured as diodes!
2. $\beta = \infty$: no I_B in any transistor

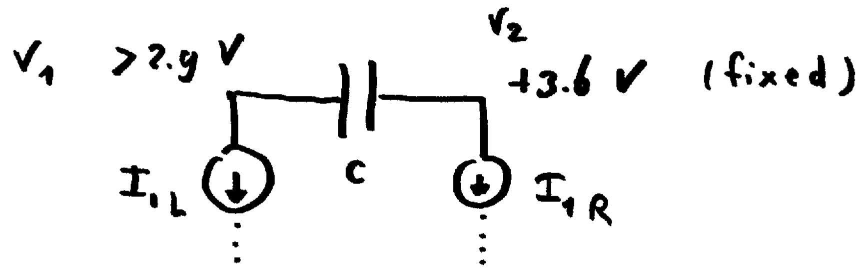
Assume : a transistor bearing current has $V_{BE} = 0.7$ V, a transistor with $V_{BE} < 0.7$ V bears no current.

The transistors Q1 and Q2 are alternatingly connected / disconnected because the voltage over the capacitor C varies.

Imagine Q1 is switched off (and thus also Q3). Assuming $V_{BE} = 0.7$ for the other transistors we have the following situation:
(V_{CC} = +5 V)



The voltage of the left side of the capacitor has to be larger than 2.9 V (because V_{E1} has to be larger than $V_{B1} - 0.7$ V).



The left current I_{1L} pulls charge out of the capacitor C, and V_1 drops. This drop is linear, because the current is constant, independent of V_1.

The voltage of V_{E1} (= V_1 of capacitor) thus keeps dropping

When V_{E1} drops below 2.9 volt, transistor Q1 opens and this induces a chain of events:

Q1 opens

Q3 opens

$V_{E3} = 4.3\ V$

$V_{E5} = V_{E3} - 0.7\ V = 3.6\ V$

$V_{B2} = V_{E5} = 3.6\ V$

Q2 closes ($V_{BE2} = 0$)

Q4 closes

$Q_{E4} = 5\ V$

$Q_{E6} = Q_{E4} - 0.7\ V = 4.3\ V$

$V_{B1} = Q_{E6} = 4.3\ V$

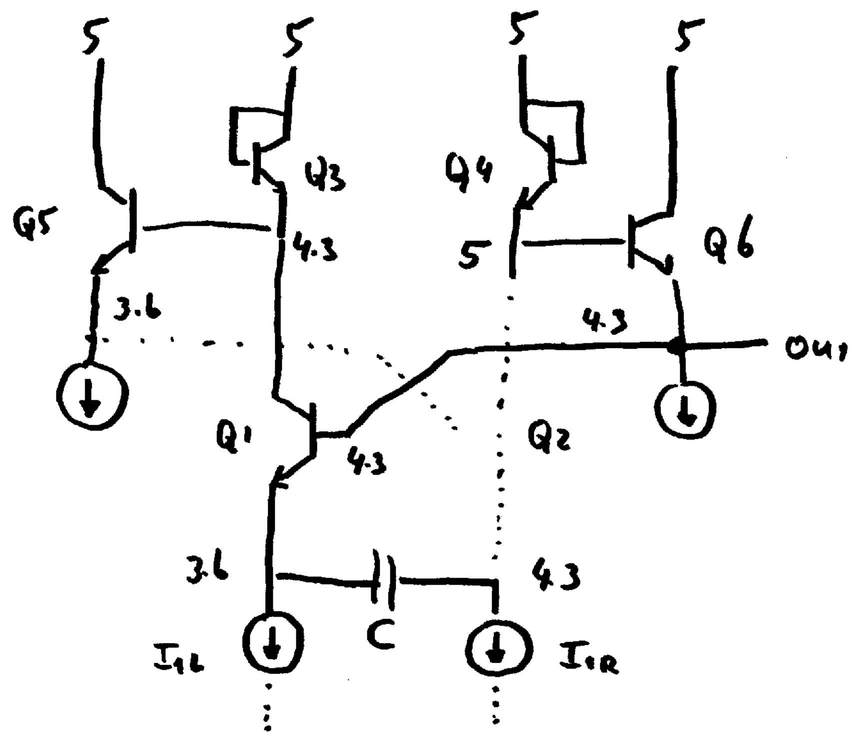
V_{E1} (was 2.9 V) jumps up to $4.3\ V - 0.7\ V = 3.6\ V$

(Q in C is constant $\Rightarrow \Delta V_c$ = constant. was $2.9 - 3.6 = -0.7$ before above step)

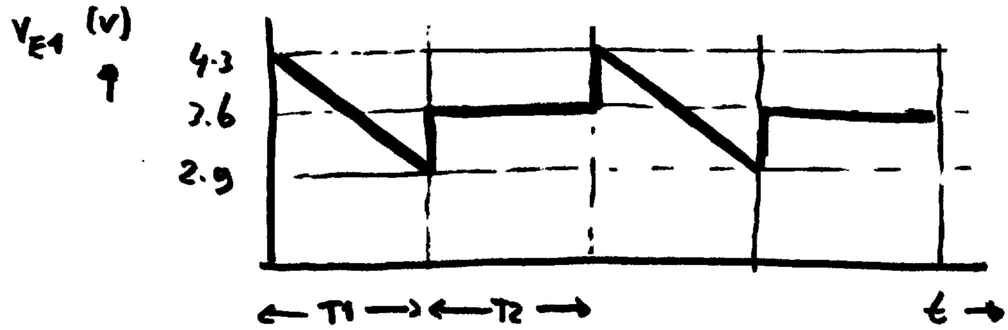
V_2 (right side of C) jumps up from 3.6 to 4.3 V

Q2 closes even more ($V_{BE2} = 3.6 - 4.3 = -0.7\ V$)

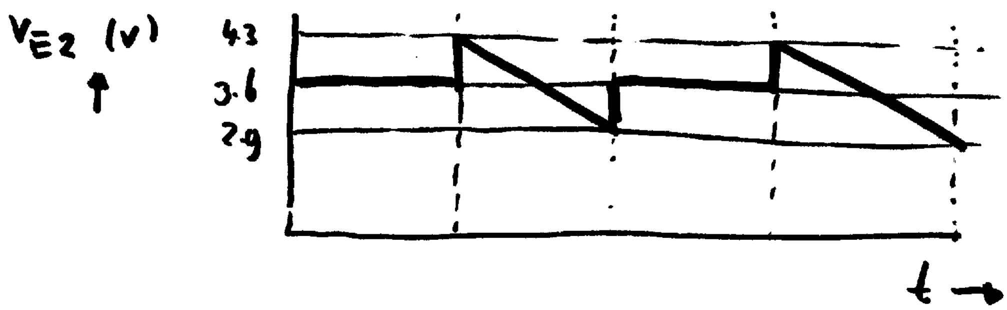
We now have the following situation:



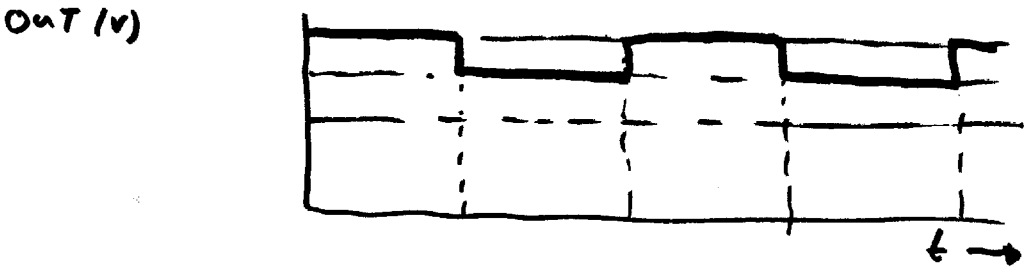
And now a 'right side' cycle starts. You get the idea.



Q1



Q2



OUT

How long does a cycle take?

T_1 is the time needed to drop the voltage at left side of C from 4.3 volt to 2.9 volt = 1.4 V

$$C \equiv Q/V$$

$$Q = CV \qquad \frac{\Delta Q}{\Delta t} = \frac{C}{\Delta t} V$$

$$\frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = I_{1L} \qquad \text{(because } I_{1L} \text{ is constant)}$$

$$\Rightarrow \quad I_{1L} = \frac{CV}{\Delta t} \Rightarrow \Delta t = \frac{CV}{I_{1L}} = \frac{C \times (1.4 \text{ volt})}{I_{1L}}$$

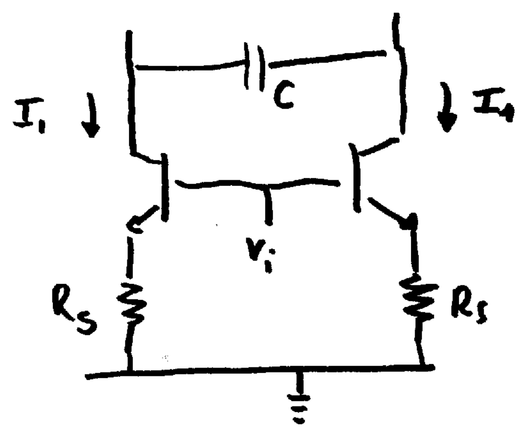
Example: $C = 2.5$ pF, $I_1 = 10\ \mu A$

$T_1 = 3.5\ 10^{-7}$ s

If $I_{1L} = I_{1R}$ then $T_2 = T_1$ and $f_{osc} = \frac{1}{2T_1}$

How to make the oscillator above a voltage-controlled oscillator?

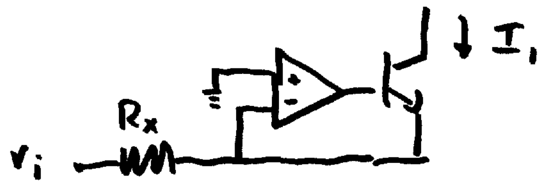
Make the current sources I_1 voltage-controlled current sources



$$I_1 = \frac{V_i - 0.7}{R_s}$$

Or use current sources of analog multipliers

(p.16 of ch.1)



$$I_1 = -\frac{V_i}{R_x}$$

We get $$T_1 = \frac{C \cdot (1.4 \text{ volt}) \cdot R_x}{V_i}$$

$$f = \frac{1}{2T_1} = \frac{1}{2 \cdot C \cdot (1.4 \text{ volt}) R_x} \times V_i$$